Exercise 3.1 (Solutions)

Question # 1(i)

$$y = x^2 - 1 \quad \ldots\ldots (i)$$

$x = 3$ & $dx = 3.02 - 3 = 0.02$

$$y + dy = (x + dx)^2 - 1$$

$$\Rightarrow dy = (x + dx)^2 - 1 - x^2 + 1$$

$$= (x + dx)^2 - x^2$$

Put $x = 3$ & $dx = 0.02$

$$dy = (3 + 0.02)^2 - (3)^2$$

$$\Rightarrow \boxed{dy = 0.1204}$$

Taking differential of (i)

$$dy = d\left(x^2 - 1\right)$$

$$\Rightarrow dy = 2x\, dx$$

Put $x = 3$ & $dx = dx = 0.02$

$$dy = 2(3)(0.02) \quad \Rightarrow \boxed{dy = 0.12}$$

Question # 1(ii)

Do yourself as above.

Question # 1(iii)

$$y = \sqrt{x} = x^{\frac{1}{2}} \quad \ldots\ldots. (i)$$

$x = 4$ & $dx = 4.41 - 4 = 0.41$

$$y + dy = (x + dx)^{\frac{1}{2}}$$

$$\Rightarrow dy = (x + dx)^{\frac{1}{2}} - x^{\frac{1}{2}}$$

Put $x = 4$ & $dx = 0.41$

$$dy = (4 + 0.41)^{\frac{1}{2}} - (4)^{\frac{1}{2}}$$

$$= 2.1 - 2 \quad \Rightarrow \boxed{dy = 0.1}$$

Taking differential of (i)

$$dy = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)dx$$

$$= \frac{1}{2}x^{-\frac{1}{2}}\, dx = \frac{1}{2x^{\frac{1}{2}}}\, dx$$

Put $x = 4$ & $dx = dx = 0.41$

$$dy = \frac{1}{2(4)^{\frac{1}{2}}}(0.41) = \frac{0.41}{4}$$

$$\Rightarrow \boxed{dy = 0.1025}$$

Question # 2(i)

$$xy + x = 4$$

Taking differential on both sides

$$d(xy) + dx = d(4)$$

$$\Rightarrow xdy + ydx + dx = 0$$

$$\Rightarrow xdy + (y+1)dx = 0$$

$$\Rightarrow xdy = -(y+1)dx$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y+1}{x}$$

$$\& \quad \frac{dx}{dy} = -\frac{x}{y+1}$$

Question # 2(ii)

Do yourself as above

Question # 2(iii)

$$x^4 + y^2 = xy^2$$

Taking differential

$$d\left(x^4\right) + d\left(y^2\right) = d\left(xy^2\right)$$

$$\Rightarrow 4x^3dx + 2ydy = x \cdot 2ydy + y^2dx$$

$$\Rightarrow 2ydy - 2xydy = y^2dx - 4x^3dx$$

$$\Rightarrow 2y(1-x)dy = \left(y^2 - 4x^3\right)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 - 4x^3}{2y(1-x)}$$

$$\& \quad \frac{dx}{dy} = \frac{2y(1-x)}{y^2 - 4x^3}$$

Question # 2(iv)

$$xy - \ln x = c$$

Taking differential

$$d(xy) - d(\ln x) = d(c)$$

$$\Rightarrow xdy + ydx - \frac{1}{x}dx = 0$$

$$\Rightarrow xdy = \frac{1}{x}dx - ydx$$

$$= \left(\frac{1}{x} - y\right)dx$$

$$\Rightarrow xdy = \left(\frac{1-xy}{x}\right)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{1-xy}{x^2}$$

$$\& \quad \frac{dx}{dy} = \frac{x^2}{1-xy}$$

Question # 3(i)

Let $y = f(x) = \sqrt[4]{x}$

where $x = 16$ and $dx = dx = 1$

Taking differential of above

$$dy = d\left(\sqrt[4]{x}\right)$$

$$= d(x)^{\frac{1}{4}}$$

$$= \frac{1}{4}x^{\frac{1}{4}-1}dx$$

$$= \frac{1}{4}x^{-\frac{3}{4}}dx$$

$$= \frac{1}{4x^{\frac{3}{4}}}dx$$

Put $x = 16$ and $dx = 1$

$$dy = \frac{1}{4(16)^{\frac{3}{4}}}(1)$$

$$= \frac{1}{4\left(2^4\right)^{\frac{3}{4}}} = \frac{1}{4(8)} = 0.03125$$

Now $f(x+dx) \approx y+dy$

$= f(x)+dy \quad \because \ y=f(x)$

$\Rightarrow \sqrt[4]{16+1} \approx \sqrt[4]{16}+0.03125$

$\Rightarrow \sqrt[4]{17} \approx \left(2^4\right)^{\frac{1}{4}}+0.03125$

$= 2+0.03125 = 2.03125$

Question # 3(ii)

Let $y=f(x)=(x)^{\frac{1}{3}}$

Where $x=8$ & $dx=dx=0.2$

Taking differential of above

$dy = d(x)^{\frac{1}{3}}$

$= \frac{1}{3}(x)^{-\frac{2}{3}}dx = \frac{1}{3x^{\frac{2}{3}}}dx$

Put $x=8$ and $dx=0.2$

$dy = \frac{1}{3(8)^{\frac{2}{3}}}(0.2)$

$= \frac{1}{3\left(2^3\right)^{\frac{2}{3}}}(0.2) = \frac{1}{3(4)}(0.2)$

$= 0.01667$

Now $f(x+dx) \approx y+dy$

$= f(x)+dy \quad \because \ y=f(x)$

$\Rightarrow (8+0.2)^{\frac{1}{3}} = (8)^{\frac{1}{3}}+0.01667$

$\Rightarrow (8.02)^3 = 2+0.01667$

$= 2.01667$

Question # 3(iii)

Let $y = f(x) = x^{\frac{1}{5}}$

Where $x=32$ & $dx=dx=-1$

Try yourself as above.

Question # 3(iv)

Let $y=f(x)=\cos x$

Where $x=30^\circ$ & $dx=-1^\circ=-\frac{p}{180}\text{ rad}$

$=-0.01745\text{ rad}$

Now $dy = d(\cos x)$

$= -\sin x\ dx$

Put $x=30^\circ$ and $dx=dx=-0.01745$

$dy = -\sin 30^\circ(-0.01745)$

$= -(0.5)(-0.01745) = 0.008725$

Now $f(x+dx) \approx y+dy$

$= f(x)+dy$

$\Rightarrow \cos(30-1) = \cos 30^\circ+0.008725$

$\Rightarrow \cos 29^\circ = 0.866+0.008725$

$= 0.8747$

Question # 3(v)

Let $y=f(x)=\sin x$

Where $x=60^\circ$ & $dx=1^\circ=\frac{p}{180}\text{ rad}$

$=0.01745\text{ rad}$

Now $dy = d(\sin x)$

$= \cos x\ dx$

Put $x=60^\circ$ and $dx=dx=0.01745$

$dy = \cos 60^\circ(0.01745)$

$= (0.5)(0.01745) = 0.008725$

Now $f(x+dx) \approx y+dy$

$= f(x)+dy$

$\Rightarrow \sin(60+1) = \sin 60^\circ+0.008725$

$\Rightarrow \sin 61^\circ = 0.866+0.008725$

$= 0.8747$

Question # 4

Let x be the length of side of cube where

$x=5$ & $dx=5.02-5=0.02$

Assume V denotes the volume of the cube.

Then $V = x\cdot x\cdot x$

$= x^3$

Taking differential

$dV = 3x^2dx$

Put $x=5$ & $dx=dx=0.02$

$dV = 3(5)^2(0.02)$

$= 1.5$

Hence increase in volume is 1.5 cubic unit.

Question # 5

Let x denotes diameter of a disc

Where $x=44\text{ cm}$ & $dx=44.4-44=0.4$

Then radius $= \frac{x}{2}$

Let A denotes the area of the disc

Then $A = p(\text{radius})^2$

$= p\left(\frac{x}{2}\right)^2 = \frac{p}{4}x^2$

Taking differential

$dA = d\left(\frac{p}{4}x^2\right)$

$= \frac{p}{4}\cdot 2x\cdot dx = \frac{p}{2}x\ dx$

Put $x=44$ and $dx=dx=0.4$

$dA = \frac{p}{2}(44)(0.4)$

$= (3.14)(22)(0.4)$

$= 27.65$

Hence change in area is 27.65 cm^2

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NOTE

If you find any mistake in these notes or you think you have easiest method of any question in these notes. Please submit it to the above email or post it or hand over at House # 143, St #6, Mehar Colony, 49-Tail, Sargodha. (: (048)3750143