Question # 1

$$\text{L.H.S} = \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25}$$

$$\because \sin^{-1}A + \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$$

$$= \sin^{-1}\left(\frac{5}{13}\sqrt{1-\left(\frac{7}{25}\right)^2} + \frac{7}{25}\sqrt{1-\left(\frac{5}{13}\right)^2}\right)$$

$$= \sin^{-1}\left(\frac{5}{13}\sqrt{1-\frac{49}{625}} + \frac{7}{25}\sqrt{1-\frac{25}{169}}\right) = \sin^{-1}\left(\frac{5}{13}\sqrt{\frac{576}{625}} + \frac{7}{25}\sqrt{\frac{144}{169}}\right)$$

$$= \sin^{-1}\left(\frac{5}{13}\left(\frac{24}{25}\right) + \frac{7}{25}\left(\frac{12}{13}\right)\right) = \sin^{-1}\left(\frac{120}{325} + \frac{84}{325}\right) = \sin^{-1}\left(\frac{204}{325}\right)$$

$$= \frac{p}{2} - \cos^{-1}\left(\frac{204}{325}\right) \qquad \because \sin^{-1}q = \frac{p}{2} - \cos^{-1}q$$

$$= \cos^{-1}(0) - \cos\left(\frac{204}{325}\right) \qquad \because \frac{p}{2} = \cos^{-1}(0)$$

$$\because \cos^{-1}A - \cos^{-1}B = \cos^{-1}\left(AB + \sqrt{(1-A^2)(1-B^2)}\right)$$

$$= \cos^{-1}\left((0)\left(\frac{204}{325}\right) + \sqrt{\left(1-(0)^2\right)\left(1-\left(\frac{204}{325}\right)^2\right)}\right)$$

$$= \cos^{-1}\left(0 + \sqrt{(1-0)\left(1-\frac{41616}{105625}\right)}\right) = \cos^{-1}\left(\sqrt{(1)\left(\frac{64009}{105625}\right)}\right)$$

$$= \cos^{-1}\left(\sqrt{\frac{64009}{105625}}\right) = \cos^{-1}\frac{253}{325} = \text{R.H.S}$$

Question # 2

$$\text{L.H.S} = \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5}$$

$$= \tan^{-1}\left(\frac{\frac{1}{4} + \frac{1}{5}}{1-\left(\frac{1}{4}\right)\left(\frac{1}{5}\right)}\right) \qquad \because \tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

$$= \tan^{-1}\left(\frac{\frac{9}{20}}{1-\frac{1}{20}}\right) = \tan^{-1}\left(\frac{\frac{9}{20}}{\frac{19}{20}}\right) = \tan^{-1}\left(\frac{9}{19}\right) = \text{R.H.S}$$

Question # 3

Suppose $\qquad a = \sin^{-1}\frac{12}{13}$ ……………… (i)

$$\Rightarrow \sin a = \frac{12}{13}$$

Now $\quad \cos a = \sqrt{1-\sin^2 a} = \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$

Now $\tan\frac{a}{2}=\sqrt{\frac{1-\cos a}{1+\cos a}}=\sqrt{\frac{1-\frac{5}{13}}{1+\frac{5}{13}}}=\sqrt{\frac{\frac{8}{13}}{\frac{18}{13}}}=\sqrt{\frac{8}{18}}=\sqrt{\frac{4}{9}}=\frac{2}{3}$

$\Rightarrow\ \frac{a}{2}=\tan^{-1}\left(\frac{2}{3}\right)\quad\Rightarrow\ a=2\tan^{-1}\frac{2}{3}$ ……………… (ii)

from (i) and (ii)

$2\tan^{-1}\frac{2}{3}=\sin^{-1}\frac{12}{13}$ *proved.*

Question # 4

Suppose $a=\tan^{-1}\frac{120}{119}$ ………………… (i)

$\Rightarrow\ \tan a=\frac{120}{119}$

Now $\sec a=\sqrt{1+\tan^2 a}$

$=\sqrt{1+\left(\frac{120}{119}\right)^2}=\sqrt{1+\frac{14400}{14161}}=\sqrt{\frac{28561}{14161}}=\frac{169}{119}$

So $\cos a=\frac{1}{\sec a}=\frac{1}{\frac{169}{119}}=\frac{119}{169}$

Now $\cos\frac{a}{2}=\sqrt{\frac{1+\cos a}{2}}=\sqrt{\frac{1+\frac{119}{169}}{2}}=\sqrt{\frac{\frac{288}{169}}{2}}=\sqrt{\frac{288}{2\times169}}=\sqrt{\frac{144}{169}}=\frac{12}{13}$

$\Rightarrow\ \frac{a}{2}=\cos^{-1}\frac{12}{13}\quad\Rightarrow\ a=2\cos^{-1}\frac{12}{13}$ ………………… (ii)

From (i) and (ii)

$\tan^{-1}\frac{120}{119}=2\cos^{-1}\frac{12}{13}$ *proved.*

Question # 5

Suppose $a=\sin^{-1}\frac{1}{\sqrt{5}}$ ………………… (i) **Correction*

$\Rightarrow\ \sin a=\frac{1}{\sqrt{5}}$

Now $\cos a=\sqrt{1-\sin^2 a}=\sqrt{1-\left(\frac{1}{\sqrt{5}}\right)^2}=\sqrt{1-\frac{1}{5}}=\sqrt{\frac{4}{5}}=\frac{2}{\sqrt{5}}$

So $\tan a=\frac{\sin a}{\cos a}=\frac{\frac{1}{\sqrt{5}}}{\frac{2}{\sqrt{5}}}=\frac{1}{2}$

$\Rightarrow\ a=\tan^{-1}\frac{1}{2}$ ………………… (ii)

From (i) and (ii)

$\sin^{-1}\frac{1}{\sqrt{5}}=\tan^{-1}\frac{1}{2}$

Now $\cot^{-1}3=\tan^{-1}\frac{1}{3}$ $\qquad\because \cot^{-1}x=\tan^{-1}\frac{1}{x}$

And $\text{L.H.S}=\sin^{-1}\frac{1}{\sqrt{5}}+\cot^{-1}3$

$=\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}$

$$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}\right) = \tan^{-1}\left(\frac{\frac{5}{6}}{1-\frac{1}{6}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) = \tan^{-1}(1) = \frac{p}{4} = \text{R.H.S}$$ *proved.*

Question # 6

$$\text{L.H.S} = \sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}$$

$$= \sin^{-1}\left(\frac{3}{5}\sqrt{1-\left(\frac{8}{17}\right)^2}+\frac{8}{17}\sqrt{1-\left(\frac{3}{5}\right)^2}\right)$$

$$= \sin^{-1}\left(\frac{3}{5}\sqrt{1-\frac{64}{289}}+\frac{8}{17}\sqrt{1-\frac{9}{25}}\right) = \sin^{-1}\left(\frac{3}{5}\sqrt{\frac{225}{289}}+\frac{8}{17}\sqrt{\frac{16}{25}}\right)$$

$$= \sin^{-1}\left(\frac{3}{5}\left(\frac{15}{17}\right)+\frac{8}{17}\left(\frac{4}{5}\right)\right) = \sin^{-1}\left(\frac{45}{85}+\frac{32}{85}\right)$$

$$= \sin^{-1}\left(\frac{77}{85}\right) = \text{R.H.S}$$ *proved.*

Question # 7

$$\text{L.H.S} = \sin^{-1}\frac{77}{85}-\sin^{-1}\frac{3}{5}$$

$$= \left(\frac{p}{2}-\cos^{-1}\frac{77}{85}\right)-\left(\frac{p}{2}-\cos^{-1}\frac{3}{5}\right) \qquad \because \sin^{-1}x = \frac{p}{2}-\cos^{-1}x$$

$$= \frac{p}{2}-\cos^{-1}\frac{77}{85}-\frac{p}{2}+\cos^{-1}\frac{3}{5} = \cos^{-1}\frac{3}{5}-\cos^{-1}\frac{77}{85}$$

$$= \cos^{-1}\left(\left(\frac{3}{5}\right)\left(\frac{77}{85}\right)+\sqrt{\left(1-\left(\frac{3}{5}\right)^2\right)\left(1-\left(\frac{77}{85}\right)^2\right)}\right)$$

$$= \cos^{-1}\left(\frac{231}{425}+\sqrt{\left(1-\frac{9}{25}\right)\left(1-\frac{5929}{7225}\right)}\right)$$

$$= \cos^{-1}\left(\frac{231}{425}+\sqrt{\left(\frac{16}{25}\right)\left(\frac{1296}{7225}\right)}\right) = \cos^{-1}\left(\frac{231}{425}+\sqrt{\frac{20736}{180625}}\right)$$

$$= \cos^{-1}\left(\frac{231}{425}+\frac{144}{425}\right) = \cos^{-1}\left(\frac{375}{425}\right) = \cos^{-1}\left(\frac{15}{17}\right) = \text{L.H.S}$$ *proved*

Question # 8

Suppose $a = \cos^{-1}\frac{63}{65}$ ………………… (i)

$$\Rightarrow \cos a = \frac{63}{65}$$

Now $\sin a = \sqrt{1-\cos^2 a} = \sqrt{1-\left(\frac{63}{65}\right)^2} = \sqrt{1-\frac{3969}{4225}} = \sqrt{\frac{256}{4225}} = \frac{16}{65}$

$\Rightarrow a = \sin^{-1}\left(\frac{16}{65}\right)$ ………………… (ii)

So from equation (i) and (ii)

$$\cos^{-1}\left(\frac{63}{65}\right) = \sin^{-1}\left(\frac{16}{65}\right)$$

Now suppose $b=\tan^{-1}\dfrac{1}{5}$ ………………… (iii)

$$\Rightarrow\ \tan b = 1/5$$

So $\sec a=\sqrt{1+\tan^2 a}=\sqrt{1+\left(\dfrac{1}{5}\right)^2}=\sqrt{1+\dfrac{1}{25}}=\sqrt{\dfrac{26}{25}}=\dfrac{\sqrt{26}}{5}$

So $\cos b=\dfrac{1}{\sec b}=\dfrac{1}{\sqrt{26}/5}=\dfrac{5}{\sqrt{26}}$

As $\dfrac{\sin b}{\cos b}=\tan b\quad\Rightarrow\ \sin b=\tan b\cdot\cos b$

$$\Rightarrow\ \sin b=\left(\frac{1}{5}\right)\left(\frac{5}{\sqrt{26}}\right)=\frac{1}{\sqrt{26}}$$

$$\Rightarrow\ b=\sin^{-1}\frac{1}{\sqrt{26}}\ \text{…………………}\ (iv)$$

From (iii) and (iv)

$$\tan^{-1}\frac{1}{5}=\sin^{-1}\frac{1}{\sqrt{26}}$$

Now L.H.S = $\cos^{-1}\dfrac{63}{65}+2\tan^{-1}\dfrac{1}{5}$

$$=\sin^{-1}\frac{16}{65}+2\sin^{-1}\frac{1}{\sqrt{26}}=\sin^{-1}\frac{16}{65}+\left(\sin^{-1}\frac{1}{\sqrt{26}}+\sin^{-1}\frac{1}{\sqrt{26}}\right)$$

$$=\sin^{-1}\frac{16}{65}+\sin^{-1}\left(\frac{1}{\sqrt{26}}\sqrt{1-\left(\frac{1}{\sqrt{26}}\right)^2}+\frac{1}{\sqrt{26}}\sqrt{1-\left(\frac{1}{\sqrt{26}}\right)^2}\right)$$

$$=\sin^{-1}\frac{16}{65}+\sin^{-1}\left(\frac{1}{\sqrt{26}}\sqrt{1-\frac{1}{26}}+\frac{1}{\sqrt{26}}\sqrt{1-\frac{1}{26}}\right)$$

$$=\sin^{-1}\frac{16}{65}+\sin^{-1}\left(\frac{1}{\sqrt{26}}\sqrt{\frac{25}{26}}+\frac{1}{\sqrt{26}}\sqrt{\frac{25}{26}}\right)$$

$$=\sin^{-1}\frac{16}{65}+\sin^{-1}\left(\frac{1}{\sqrt{26}}\cdot\frac{5}{\sqrt{26}}+\frac{1}{\sqrt{26}}\cdot\frac{5}{\sqrt{26}}\right)$$

$$=\sin^{-1}\frac{16}{65}+\sin^{-1}\left(\frac{5}{26}+\frac{5}{26}\right)=\sin^{-1}\frac{16}{65}+\sin^{-1}\left(\frac{5}{13}\right)$$

$$=\sin^{-1}\left(\frac{16}{65}\sqrt{1-\left(\frac{5}{13}\right)^2}+\frac{5}{13}\sqrt{1-\left(\frac{16}{65}\right)^2}\right)$$

$$=\sin^{-1}\left(\frac{16}{65}\sqrt{1-\frac{25}{169}}+\frac{5}{13}\sqrt{1-\frac{256}{4225}}\right)$$

$$=\sin^{-1}\left(\frac{16}{65}\sqrt{1-\frac{25}{169}}+\frac{5}{13}\sqrt{1-\frac{256}{4225}}\right)=\sin^{-1}\left(\frac{16}{65}\sqrt{\frac{144}{169}}+\frac{5}{13}\sqrt{\frac{3969}{4225}}\right)$$

$$=\sin^{-1}\left(\frac{16}{65}\sqrt{\frac{144}{169}}+\frac{5}{13}\sqrt{\frac{3969}{4225}}\right)=\sin^{-1}\left(\frac{16}{65}\left(\frac{12}{13}\right)+\frac{5}{13}\left(\frac{63}{65}\right)\right)$$

$$=\sin^{-1}\left(\frac{192}{845}+\frac{315}{845}\right)=\sin^{-1}\left(\frac{3}{5}\right)=\text{R.H.S}$$ *proved*

Question # 9

$$\text{L.H.S} = \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19}$$ **Correction*

$$= \tan^{-1}\left(\frac{3/4 + 3/5}{1-\left(3/4\right)\left(3/5\right)}\right) - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\left(\frac{27/20}{1-9/20}\right) - \tan^{-1}\frac{8}{19} = \tan^{-1}\left(\frac{27/20}{11/20}\right) - \tan^{-1}\frac{8}{19} = \tan^{-1}\left(\frac{27}{11}\right) - \tan^{-1}\left(\frac{8}{19}\right)$$

$$= \tan^{-1}\left(\frac{27/11 - 8/19}{1+\left(27/11\right)\left(8/19\right)}\right) = \tan^{-1}\left(\frac{425/209}{1+216/209}\right) = \tan^{-1}\left(\frac{425/209}{1+216/209}\right)$$

$$= \tan^{-1}\left(\frac{425/209}{425/209}\right) = \tan^{-1}(1) = \frac{p}{4} = \text{R.H.S}$$ *proved.*

Question # 10 *Do Yourself*

Question # 11

$$\text{L.H.S} = \tan^{-1}\frac{1}{11} + \tan^{-1}\frac{5}{6}$$

= *Solve this* ……………….. (i)

$$\text{R.H.S} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$$

= *Solve this* ………………. (ii)

From (i) and (ii)

L.H.S = R.H.S

Question # 12

$$\text{L.H.S} = 2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$$

Now do yourself as Question # 9.

Question # 13

Suppose $y = \sin^{-1} x$

$\Rightarrow \sin y = x$

Since $\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-x^2}$

$\Rightarrow \cos\left(\sin^{-1} x\right) = \sqrt{1-x^2}$ *Proved*

Question # 14

Suppose $y = \cos^{-1} x$

Then $\cos y = x$

Also $\sin y = \sqrt{1-\cos^2 y} = \sqrt{1-x^2}$

Now $\sin\left(2\cos^{-1} x\right) = \sin(2y)$

$$= 2\sin y \cdot \cos y$$

$$= 2\sqrt{1-x^2} \cdot x$$

$$= 2x\sqrt{1-x^2}$$ *Proved*

Question # 15

Suppose $y = \sin^{-1} x \quad \Rightarrow \quad \sin y = x$

& $\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$

Now $\cos\left(2\sin^{-1} x\right) = \cos 2y$

$= \cos^2 y - \sin^2 y$

$= \left(\sqrt{1 - x^2}\right)^2 - x^2 = 1 - x^2 - x^2$

$= 1 - 2x^2$ *Proved*

Question # 16

Suppose $y = \tan^{-1}(-x)$ …………….. (i)

$\Rightarrow \tan y = -x \quad \Rightarrow \quad -\tan y = x$

$\Rightarrow \tan(-y) = x$ $\qquad \because \; -\tan q = \tan(-q)$

$\Rightarrow -y = \tan^{-1} x$

$\Rightarrow y = -\tan^{-1} x$ ……………… (ii)

From equation (i) and (ii)

$\tan^{-1}(-x) = -\tan^{-1} x$ *Proved*

Question # 17 *Do yourself as above*

Question # 18

Suppose $y = p - \cos^{-1} x$ ………………… (i)

$\Rightarrow p - y = \cos^{-1} x \quad \Rightarrow \quad \cos(p - y) = x$

$\Rightarrow \cos p \cos y + \sin p \sin y) = x \quad \Rightarrow \quad (-1)\cos y + (0)\sin y) = x$

$\Rightarrow -\cos y + 0 = x \quad \Rightarrow \quad -\cos y = x$

$\Rightarrow \cos y = -x \quad \Rightarrow \quad y = \cos^{-1}(-x)$ ………… (ii)

From (i) and (ii)

$\cos^{-1}(-x) = p - \cos^{-1} x$ *Proved*

Question # 19

Suppose $y = \sin^{-1} x \quad \Rightarrow \quad \sin y = x$

Now $\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$

& $\tan y = \dfrac{\sin y}{\cos y} = \dfrac{x}{\sqrt{1 - x^2}}$

Now $\tan\left(\sin^{-1} x\right) = \tan y = \dfrac{x}{\sqrt{1 - x^2}}$ *proved.*

Question # 20

Since $x = \sin^{-1}\dfrac{1}{2} \quad \Rightarrow \quad \sin x = \dfrac{1}{2}$

Now $\cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\tan x = \dfrac{\sin x}{\cos x} = \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{1}{\sqrt{3}}$, $\qquad \cot x = \dfrac{1}{\tan x} = \dfrac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$

$\sec x = \dfrac{1}{\cos x} = \dfrac{1}{\frac{\sqrt{3}}{2}} = \dfrac{2}{\sqrt{3}}$, $\qquad \operatorname{cosec} x = \dfrac{1}{\sin x} = \dfrac{1}{\frac{1}{2}} = 2$

The End