Chapter # 2

Logarithms

Exercise # 2.1

Question # 1: Express the following numbers in scientific notation.

(i) $\mathbf{2000000}$
$= 2 \times 10^6$ (Answer)

(ii) $\mathbf{48900}$
$= 4.89 \times 10^4$ (Answer)

(iii) $\mathbf{0.0042}$
$= 4.2 \times 10^{-3}$ (Answer)

(iv) $\mathbf{0.0000009}$
$= 9 \times 10^{-7}$ (Answer)

(v) $\mathbf{73 \times 10^3}$
$= 7.3 \times 10^1 \times 10^3$
$= 7.3 \times 10^{1+3}$
$= 7.3 \times 10^4$ (Answer)

(vi) $\mathbf{0.65 \times 10^2}$
$= 6.5 \times 10^{-1} \times 10^2$
$= 6.5 \times 10^{-1+2}$
$= 6.5 \times 10^1$ (Answer)

Question # 2: Express the following numbers in ordinary notation.

(i) $\mathbf{8.04 \times 10^2}$
$= 804$ (Answer)

(ii) $\mathbf{3 \times 10^5}$
$= 300000$ (Answer)

(iii) $\mathbf{1.5 \times 10^{-2}}$
$= 0.015$ (Answer)

(iv) $\mathbf{1.77 \times 10^7}$
$= 17700000$ (Answer)

(v) $\mathbf{5.5 \times 10^{-6}}$
$= 0.0000055$ (Answer)

(vi) $\mathbf{4 \times 10^{-5}}$
$= 0.00004$ (Answer)

Question # 3: The speed of light is approximately 3×10^8 meters per second. Express it in standard form.

Speed of light $= 3 \times 10^8\ ms^{-1}$

In standard form: $300000000\ ms^{-1}$ (Answer)

Question # 4: The circumference of the Earth at the equator is about 4007500 meters. Express this number in scientific notation.

Circumference of Earth $= 40075000\ m$

In scientific notation: $4.0075 \times 10^7\ m$ (Answer)

Question # 5: The diameter of Mars is 6.779×10^3 km. Express this number in standard form.

Diameter of Mars $= 6.779 \times 10^3\ km$

In standard form: $6779\ km$ (Answer)

Question # 6: The diameter of Earth is about 1.2756×10^4 km. Express this number in standard form.

Diameter of Earth $= 1.2756 \times 10^4\ km$

In standard form: $12756\ km$ (Answer)

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Exercise # 2.2

Question # 1: Express each of the following in logarithmic form.

(i) $10^3 = 1000$

$\log_{10} 1000 = 3$ (Ans)

(ii) $2^8 = 256$

$\log_2 256 = 8$ (Ans)

(iii) $3^{-3} = \frac{1}{27}$

$\log_3 \frac{1}{27} = -3$ (Ans)

(iv) $20^2 = 400$

$\log_{20} 400 = 2$ (Ans)

(v) $16^{-\frac{1}{4}} = \frac{1}{2}$

$\log_{16} \frac{1}{2} = -\frac{1}{4}$ (Ans)

(vi) $11^2 = 121$

$\log_{11} 121 = 2$ (Ans)

(vii) $p = q^r$

$q^r = p$

$\log_q p = r$ (Ans)

(viii) $(32)^{\frac{-1}{5}} = \frac{1}{2}$

$\log_{32} \frac{1}{2} = \frac{-1}{5}$ (Ans)

Question # 2: Express each of the following in exponential form.

(i) $\log_5 125 = 3$

$5^3 = 125$ (Ans)

(ii) $\log_2 16 = 4$

$2^4 = 16$ (Ans)

(iii) $\log_{23} 1 = 0$

$23^0 = 1$ (Ans)

(iv) $\log_5 5 = 1$

$5^1 = 5$ (Ans)

(v) $\log_2 \frac{1}{8} = -3$

$2^{-3} = \frac{1}{8}$ (Ans)

(vi) $\frac{1}{2} = \log_9 3$

$\log_9 3 = \frac{1}{2}$

$9^{\frac{1}{2}} = 3$ (Ans)

(vii) $5 = \log_{10} 100000$

$\log_{10} 100000 = 5$

$10^5 = 100000$ (Ans)

(viii) $\log_4 \frac{1}{16} = -2$

$4^{-2} = \frac{1}{16}$ (Ans)

Question # 3: Find the value of x in each of the following.

(i) $\log_x 64 = 3$

$x^3 = 64$

$x^{\cancel{3}} = 4^{\cancel{3}}$

$x = 4$ (Ans)

4	64
4	16
4	4
	1

(ii) $\log_5 1 = x$

$5^x = 1$

$\cancel{5}^x = \cancel{5}^0$

$x = 0$ (Ans)

(iii) $\log_x 8 = 1$

$x^1 = 8$

$x = 8$ (Ans)

(iv) $\log_{10} x = -3$

$10^{-3} = x$

OR,

$x = 10^{-3}$

$x = \frac{1}{10^3}$

$x = \frac{1}{1000}$ (Ans)

(v) $\log_4 x = \frac{3}{2}$

$4^{\frac{3}{2}} = x$

OR,

$x = 4^{\frac{3}{2}}$

$x = 2^{\cancel{2} \times \frac{3}{\cancel{2}}}$

$x = 2^3$

$x = 8$ (Ans)

(vi) $\log_2 1024 = x$

$2^x = 1024$

$\cancel{2}^x = \cancel{2}^{10}$

$x = 10$ (Ans)

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

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Exercise # 2.3

Question # 1: Find characteristic of the following numbers.

Characteristic is power of 10 when a number is in scientific notation
Mantissa is 'log' of coefficient/decimal part, of number in scientific notation

Rough Work

(i) $\mathbf{5287}$

Characteristic $= 3$ (Answer) | 5.287×10^3

(ii) $\mathbf{59.28}$

Characteristic $= 1$ (Answer) | 5.928×10^1

(iii) $\mathbf{0.0567}$

Characteristic $= -2$ (Answer) | 5.67×10^{-2}

(iv) $\mathbf{234.7}$

Characteristic $= 2$ (Answer) | 2.347×10^2

(v) $\mathbf{0.000049}$

Characteristic $= -5$ (Answer) | 4.9×10^{-5}

(vi) $\mathbf{145000}$

Characteristic $= 5$ (Answer) | 1.45×10^5

Question # 2: Find logarithm of the following numbers.

$$\because \log(Number) = Characteristic + Mantissa$$

Rough Work

(i) $\mathbf{43}$

Characteristic $= 1$ Mantissa $= 0.6335$ | 4.3×10^1

$\log 43 = 1 + 0.6335$

$\log 43 = 1.6335$ (Answer)

(ii) $\mathbf{579}$

Characteristic $= 2$ Mantissa $= 0.7627$ | 5.79×10^2

$\log 579 = 2 + 0.7627$

$\log 579 = 2.7627$ (Answer)

(iii) $\mathbf{1.982}$

Characteristic $= 0$ Mantissa $= 0.2971$ | 1.982×10^0

$\log 1.982 = 0 + 0.2971$

$\log 1.982 = 0.2971$ (Answer)

(iv) $\mathbf{0.0876}$

Characteristic $= -2$ Mantissa $= 0.9425$ | 8.76×10^{-2}

$\log 0.0876 = -2 + 0.9425$

$\log 0.0876 = -1.0575$ (Answer)

(v) $\mathbf{0.047}$

Characteristic $= -2$ Mantissa $= 0.6721$ | 4.7×10^{-2}

$\log 0.047 = -2 + 0.6721$

$\log 0.047 = -1.3279$ (Answer)

(vi) $\mathbf{0.000354}$

Characteristic $= -4$ Mantissa $= 0.5490$

$\log 0.000354 = -4 + 0.5490$

$\log 0.000354 = -3.4510$ (Answer)

3.54×10^{-4}

Question # 3: If $log 3.177 = 0.5019$, then find:

Rough Work

(i) $\mathbf{\log 3177}$

Characteristic $= 3$ Mantissa $= 0.5019$

$\log 3177 = 3 + 0.5019$

$\log 3177 = 3.5019$ (Answer)

3.177×10^{3}

(ii) $\mathbf{\log 31.77}$

Characteristic $= 1$ Mantissa $= 0.5019$

$\log 31.77 = 1 + 0.5019$

$\log 31.77 = 1.5019$ (Answer)

3.177×10^{1}

(iii) $\mathbf{\log 0.03177}$

Characteristic $= -2$ Mantissa $= 0.5019$

$\log 0.03177 = -2 + 0.5019$

$\log 0.03177 = -1.4981$ (Answer)

3.177×10^{-2}

Question # 4: Find the value of x.

(i) $\mathbf{\log x = 0.0065}$

Characteristic $= 0$ Mantissa $= 0.0065$

$x = antilog\ (0.0065)$

$x = 1.015$ (Answer)

(ii) $\mathbf{\log x = 1.192}$

Characteristic $= 1$ Mantissa $= 0.192$

$x = antilog\ (0.192)$

$x = 1.556$ $(Due\ to\ Charac = 1)$

$x = 15.56$ (Answer)

(iii) $\mathbf{\log x = -3.434}$

Adding and subtracting '4'

$\log x = -4 + 4 - 3.434$

$\log x = -4 + 0.566$

Characteristic $= -4$ Mantissa $= 0.566$

$x = antilog\ (0.566)$

$x = 3.6813$ $(Due\ to\ Charac = -4)$

$x = 0.0003681$ (Answer)

(iv) $\mathbf{\log x = -1.5726}$

Adding and subtracting '2'

$\log x = -2 + 2 - 1.5726$

$\log x = -2 + 0.4274$

Characteristic $= -2$ Mantissa $= 0.4274$

$x = antilog\ (0.4274)$

$x = 2.6755$ $(Due\ to\ Charac = -2)$
$x = 0.02675$ (Answer)

(v) $\mathbf{\log x = 4.3561}$
Characteristic $= 4$ Mantissa $= 0.3561$
$x = antilog\ (0.3561)$
$x = 2.2704$ $(Due\ to\ Charac = 4)$
$x = 22704$ (Answer)

(vi) $\mathbf{\log x = -2.0184}$
Adding and subtracting '3'
$\log x = -3 + 3 - 2.0184$
$\log x = -3 + 0.9816$
Characteristic $= -3$ Mantissa $= 0.9816$
$x = antilog\ (0.9816)$
$x = 9.5852$ $(Due\ to\ Charac = -3)$
$x = 0.009585$ (Answer)

Alternative Method (Question # 4)

(i) $\mathbf{\log x = 0.0065}$
Taking 'Antilog' on both sides
$Antilog\ logx = Antilog\ (0.0065)$
$x = 1.015$

(ii) $\mathbf{\log x = 1.192}$
Taking 'Antilog' on both sides
$Antilog\ logx = Antilog\ (1.192)$
$x = 15.56$

(iii) $\mathbf{\log x = -3.434}$
Taking 'Antilog' on both sides
$Antilog\ logx = Antilog\ (-3.434)$
$x = 0.0003681$

(iv) $\mathbf{\log x = -1.5726}$
Taking 'Antilog' on both sides
$Antilog\ logx = Antilog\ (-1.5726)$
$x = 0.02675$

(v) $\mathbf{\log x = 4.3561}$
Taking 'Antilog' on both sides
$Antilog\ logx = Antilog\ (4.3561)$
$x = 22704$

(vi) $\mathbf{\log x = -2.0184}$
Taking 'Antilog' on both sides
$Antilog\ logx = Antilog\ (-2.0184)$
$x = 0.009585$

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Exercise # 2.4

Question # 1: Without using calculator, evaluate the following:

(i). $\log_2 18 - \log_2 9$

$= \log_2(2\times 9) - \log_2 9$

$= \log_2 2 + \cancel{\log_2 9} - \cancel{\log_2 9}$

$= \log_2 2$ $\quad \because \log_a a = 1$

$= 1$ (Answer)

(ii). $\log_2 64 + \log_2 2$

2	64
2	32
2	16
2	8
2	4
2	2
	1

$= \log_2 2^6 + \log_2 2$

$= 6\log_2 2 + \log_2 2$ $\quad \because \log_a a = 1$

$= 6(1) + 1$

$= 6 + 1 = 7$ (Answer)

(iii). $\frac{1}{3}\log_3 8 - \log_3 18$

2	18
3	9
3	3
	1

$= \frac{1}{3}\log_3 2^3 - \log_3(2\times 3^2)$

$= \log_3(2^{\cancel{3}})^{\frac{1}{\cancel{3}}} - (\log_3 2 + \log_3 3^2)$

$= \cancel{\log_3 2} - \cancel{\log_3 2} - 2\log_3 3$

$= -2\log_3 3$ $\quad \because \log_a a = 1$

$= -2(1) = -2$ (Answer)

(iv). $2\log 2 + \log 25$

$= \log 2^2 + \log 25$

$= \log 4 + \log 25$

$= \log 4\times 25$

$= \log 100$

$= \log 10^2$

$= 2\log 10$ $\quad \because \log_{10} 10 = 1$

$= 2(1) = 2$ (Answer)

(v). $\frac{1}{3}\log_4 64 + 2\log_5 25$

4	64
4	16
4	4
	1

$= \frac{1}{3}\log_4 4^3 + 2\log_5 5^2$

$= \log_4(4^{\cancel{3}})^{\frac{1}{\cancel{3}}} + 2\times 2\log_5 5$

$= \log_4 4 + 4\log_5 5$ $\quad \because \log_a a = 1$

$= 1 + 4(1)$

$= 1 + 4 = 5$ (Answer)

(vi). $\log_3 12 + \log_3 0.25$

$= \log_3(3\times 4) + \log_3\left(\frac{\cancel{25}^{\,1}}{\cancel{100}_{\,4}}\right)$

$= \log_3 3 + \log_3 4 + \log_3\left(\frac{1}{4}\right)$ $\quad \because \log_a a = 1$

$= 1 + \cancel{\log_3 4} + \log_3 1 - \cancel{\log_3 4}$

$= 1 + \log_3 1$ $\quad \because \log_a 1 = 0$

$= 1 + 0 = 1$ (Answer)

Question # 2: Write the following as a single logarithm:

(i). $\frac{1}{2}\log 25 + 2\log 3$

$= \log(25)^{\frac{1}{2}} + \log 3^2$

$= \log(\sqrt{25}\times 9)$

$= \log(5\times 9)$

$= \log 45$ (Answer)

(ii). $\log 9 - \log\frac{1}{3}$

$= \log\frac{9}{1/3}$

$= \log\frac{9\times 3}{1}$

$= \log 27$ (Answer)

(iii). $\log_5 b^2 . \log_a 5^3$

$= 2\log_5 b \times 3\log_a 5$ $\quad \because \log_b x = \frac{\log_a x}{\log_a b}$

$= 2\times 3\left(\frac{\log b}{\cancel{\log 5}}\times\frac{\cancel{\log 5}}{\log a}\right)$

$= 6\left(\frac{\log b}{\log a}\right)$ $\quad \because \frac{\log_a x}{\log_a b} = \log_b x$

$= 6\log_a b$ (Answer)

(iv). $2\log_3 x + \log_3 y$

$= \log_3 x^2 + \log_3 y$

$= \frac{\log x^2}{\log 3} + \frac{\log y}{\log 3}$

$= \frac{\log x^2 + \log y}{\log 3}$

$= \frac{\log x^2 y}{\log 3}$ $\quad \because \frac{\log_a x}{\log_a b} = \log_b x$

$= \log_3 x^2 y$ (Answer)

(v). $\mathbf{4\log_5 x - \log_5 y + \log_5 z}$

$= \log_5 x^4 + \log_5 z - \log_5 y$

$= \log_5 \frac{x^4 z}{y}$ (Answer)

(vi). $\mathbf{2\ln a + 3\ln b - 4\ln c}$

$= \ln a^2 + \ln b^3 - \ln c^4$

$= \ln \frac{a^2 b^3}{c^4}$ (Answer)

Question # 3: Expand the following using laws of logarithms:

(i). $\mathbf{\log\left(\frac{11}{5}\right)}$

$= \log 11 - \log 5$ (Answer)

(ii). $\mathbf{\log_5 \sqrt{8a^6}}$

$= \log_5 (2^3 a^6)^{\frac{1}{2}}$

$= \log_5 \left(2^{3\times\frac{1}{2}}\, a^{6\times\frac{1}{2}}\right)$

$= \log_5 2^{\frac{3}{2}}\ a^3$

$= \log_5 2^{\frac{3}{2}} + \log_5 a^3$

$= \frac{3}{2}\log_5 2 + 3\log_5 a$ (Answer)

(iii). $\mathbf{\ln\left(\frac{a^2 b}{c}\right)}$

$= \ln a^2 + \ln b - \ln c$

$= 2\ln a + \ln b - \ln c$ (Answer)

(iv). $\mathbf{\log\left(\frac{xy}{z}\right)^{\frac{1}{9}}}$

$= \frac{1}{9}\left[\log\left(\frac{xy}{z}\right)\right]$

$= \frac{1}{9}(\log x + \log y - \log z)$ (Answer)

(v). $\mathbf{\ln \sqrt[3]{16x^3}}$

2	16
2	8
2	4
2	2
	1

$= \ln(2^4 x^3)^{\frac{1}{3}}$

$= \ln 2^{4\times\frac{1}{3}}\, x^{3\times\frac{1}{3}}$

$= \ln 2^{\frac{4}{3}}\, x$

$= \ln 2^{\frac{4}{3}} + \ln x$

$= \frac{4}{3}\ln 2 + \ln x$ (Answer)

(vi). $\mathbf{\log_2\left(\frac{1-a}{b}\right)^5}$

$= 5\log_2\left(\frac{1-a}{b}\right)$

$= 5[\log_2(1-a) - \log_2 b]$ (Answer)

Question # 4: Evaluate the value of 'x' in the following equations:

(i). $\mathbf{\log 2 + \log x = 1}$

$\log 2x = 1$

$10^1 = 2x$

$\frac{10}{2} = x$

$x = 5$ (Answer)

(ii). $\mathbf{\log_2 x + \log_2 8 = 5}$

$\log_2 8x = 5$

$2^5 = 8x$

$32 = 8x$

$\frac{32}{8} = x$

$x = 4$ (Answer)

(iii). $\mathbf{(81)^x = (243)^{x+2}}$

3	243
3	81
3	27
3	9
3	3
	1

$(3^4)^x = (3^5)^{x+2}$

$3^{4x} = 3^{5x+10}$

$4x = 5x + 10$

$4x - 5x = 10$

$-x = 10$

$x = -10$ (Answer)

(iv). $\mathbf{\left(\frac{1}{27}\right)^{x-6} = 27}$

$(27^{-1})^{x-6} = 27$

$27^{-x+6} = 27^1$

$-x + 6 = 1$

$-x = 1 - 6$

$-x = -5$

$x = 5$ (Answer)

(v). $\mathbf{\log(5x - 10) = 2}$

$10^2 = 5x - 10$

(vi). $\mathbf{\log_2(x+1) - \log_2(x-4) = 2}$

$$100 = 5x - 10$$
$$100 + 10 = 5x$$
$$110 = 5x$$
$$\frac{110}{5} = x$$
$x = 22$ (Answer)

$$\log_2\left(\frac{x+1}{x-4}\right) = 2$$
$$2^2 = \frac{x+1}{x-4}$$
$$4(x-4) = x+1$$
$$4x - 16 = x + 1$$
$$4x - x = 1 + 16$$
$$3x = 17$$
$$x = \frac{17}{3}$$
$x = 5\frac{2}{3}$ (Answer)

Question # 5: Find the value of the following with the help of logarithm table:

(i). $\frac{3.68 \times 4.21}{5.234}$

Let,

$$x = \frac{3.68 \times 4.21}{5.234}$$

Taking 'log' on both sides

$$logx = log\left(\frac{3.68 \times 4.21}{5.234}\right)$$
$$logx = log3.68 + log4.21 - log5.234$$
$$logx = 0.5658 + 0.6243 - 0.7188$$
$$logx = 0.4713$$

Taking '$antilog$' on both sides

$$antilog \times logx = anti\log 0.4713$$
$x = 2.9601$ (Answer)

(ii). $4.67 \times 2.11 \times 2.397$

Let,

$$x = 4.67 \times 2.11 \times 2.397$$

Taking 'log' on both sides

$$logx = \log(4.67 \times 2.11 \times 2.397)$$
$$logx = log4.67 + log2.11 + log2.397$$
$$logx = 0.6693 + 0.3243 + 0.3797$$
$$logx = 1.3733$$

Taking '$antilog$' on both sides

$$antilog \times logx = antilog1.3733$$
$x = 23.6194$ (Answer)

(iii). $\frac{(20.46)^2 \times (2.4122)}{754.3}$

Let,

$$x = \frac{(20.46)^2 \times (2.4122)}{754.3}$$

Taking 'log' on both sides

$$logx = log\left[\frac{(20.46)^2 \times (2.4122)}{754.3}\right]$$
$$logx = 2log20.46 + log2.4122 - log754.3$$
$$logx = 2(1.3109) + 0.3824 - 2.8775$$
$$logx = 2.6218 + 0.3824 - 2.8775$$
$$logx = 0.1267$$

Taking '$antilog$' on both sides

$$antilog \times logx = antilog0.1267$$
$x = 1.3388$ (Answer)

(iv). $\frac{\sqrt[3]{9.364} \times 21.64}{3.21}$

Let,

$$x = \frac{(9.364)^{1/3} \times 21.64}{3.21}$$

Taking 'log' on both sides

$$logx = log\left[\frac{(9.364)^{1/3} \times 21.64}{3.21}\right]$$
$$logx = \frac{1}{3}log9.364 + log21.64 - log3.21$$
$$logx = \frac{1}{3}(0.9715) + 1.335 - 0.5065$$
$$logx = 0.3238 + 1.335 - 0.5065$$
$$logx = 1.1523$$

Taking '$antilog$' on both sides

$$antilog \times logx = antilog1.1523$$
$x = 14.2003$ (Answer)

Question # 6: The formula to measure the magnitude of earthquakes is given by $M = \log_{10}\left(\frac{A}{A_o}\right)$. If amplitude ($A$) is 10,000 and reference amplitude (A_o) is 10. What is the magnitude of the earthquake?

$$M = \log_{10}\left(\frac{A}{A_o}\right)$$
$$\text{put } A = 10000, A_o = 10$$
$$M = \log_{10}\left(\frac{10000}{10}\right)$$
$$M = \log_{10}(1000)$$
$$M = \log_{10} 10^3$$
$$M = 3\log_{10} 10 \qquad \because \log_a a = 1$$
$$M = 3(1)$$
$$M = 3 \qquad \text{(Answer)}$$

Question # 7: Abdullah invested Rs. 1,00,000 in a saving scheme and gains interest at the rate of 5% per annum so that the total value of this investment after t years is Rs y. This is modelled by an equation $y = 1,00,000(1.05)^t, t \geq 0$. Find after how many years the investment will be doubled?

$$\text{Investment Amount} = 100000 \text{ Rs}$$
$$\text{Rate} = 5\%$$
$$\text{Double Investment Year} = y = 200000 \text{ Rs}$$
$$\text{Time} = t = ?$$
$$\because y = 100000(1.05)^t$$
$$200000 = 100000(1.05)^t$$
$$\frac{200000}{100000} = (1.05)^t$$
$$2 = (1.05)^t$$
$$\text{Taking '}log\text{' on both sides}$$
$$\log 2 = \log(1.05)^t$$
$$0.3010 = t(log 1.05)$$
$$0.3010 = t(0.0212)$$
$$t = \frac{0.3010}{0.0212}$$
$$t = 14 \text{ years} \qquad \text{(Answer)}$$

Question # 8: Huria is hiking up a mountain where the temperature (T) decreases by 3% (or a factor of 0.97) for every 100 meters gained in altitude. The initial temperature (T_i) at sea level is 20° C. Using the formula $T = T_i \times 0.97^{\frac{h}{100}}$, calculate the temperature at an altitude (h) of 500 meters.

Initial Temperature $= T_i = 20^oC$

Altitude $= h = 500m$

Final Temperature $= T = ?$

$$\because T = T_i \times (0.97)^{h/100}$$

$$T = 20 \times (0.97)^{500/100} \qquad \because h = 500$$

$$T = 20 \times (0.97)^5$$

Taking 'log' on both sides

$$\log T = log[20 \times (0.97)^5]$$

$$\log T = log20 + 5log0.97$$

$$\log T = 1.3010 + 5(-0.0132)$$

$$logT = 1.3010 - 0.066$$

$$logT = 1.235$$

Taking '$antilog$' on both sides

$$antilog \times logT = antilog1.235$$

$$T = 17.18^oC$$ (Answer)

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Review Exercise # 2

Question # 1: Four options are given against each statement. Encircle the correct option.

#	Answer	#	Answer
i	C	vi	C
ii	B	vii	D
iii	B	viii	C
iv	D	ix	D
v	A	x	C

Question # 2: Express the following numbers in scientific notation:

(i). $\mathbf{0.000567}$
$= 5.67 \times 10^{-4}$ (Answer)

(ii). $\mathbf{734}$
$= 7.34 \times 10^{2}$ (Answer)

(iii). $\mathbf{0.33 \times 10^3}$
$= 3.3 \times 10^{3-1}$
$= 3.3 \times 10^{2}$ (Answer)

Question # 3: Express the following numbers in ordinary notation:

(i). $\mathbf{2.6 \times 10^3}$
$= 2600$ (Answer)

(ii). $\mathbf{8.794 \times 10^{-4}}$
$= 0.0008794$ (Answer)

(iii). $\mathbf{6 \times 10^{-6}}$
$= 0.000006$ (Answer)

Question # 4: Express each of the following in logarithmic form:

(i). $\mathbf{3^7 = 2187}$
$\log_3 2187 = 7$ (Answer)

(ii). $\boldsymbol{a^b = c}$
$\log_a c = b$ (Answer)

(iii). $\mathbf{(12)^2 = 144}$
$\log_{12} 144 = 2$ (Answer)

Question # 5: Express each of the following in exponential form:

(i). $\mathbf{\log_4 8 = x}$
$4^x = 8$ (Answer)

(ii). $\mathbf{\log_9 729 = 3}$
$9^3 = 729$ (Answer)

(iii). $\mathbf{\log_4 1024 = 5}$
$4^5 = 1024$ (Answer)

Question # 6: Find the value of 'x' in the following:

(i). $\mathbf{\log_9 x = 0.5}$
$9^{0.5} = x$
$x = 9^{1/2}$
$x = (3^{\not 2})^{1/\not 2}$
$x = 3$ (Answer)

(ii). $\mathbf{\left(\frac{1}{9}\right)^{3x} = 27}$
$(9^{-1})^{3x} = 27$
$(3^{-2})^{3x} = 3^3$
$3^{-6x} = 3^3$
$-6x = 3$
$x = \frac{\not 3\ 1}{-\not 6\ 2}$
$x = -\frac{1}{2}$ (Answer)

3	27
3	9
3	3
	1

(iii). $\mathbf{\left(\frac{1}{32}\right)^{2x} = 64}$
$(32^{-1})^{2x} = 64$
$(2^{-5})^{2x} = 2^6$
$2^{-10x} = 2^6$
$-10x = 6$
$x = \frac{\not 6\ 3}{-\not{10}\ 5}$
$x = -\frac{3}{5}$ (Answer)

2	64
2	32
2	16
2	8
2	4
2	2
	1

Question # 7: Write the following as a single logarithm:

(i). $\mathbf{7\log x - 3\log y^2}$
$= \log x^7 - \log y^6$

(ii). $\mathbf{3\log 4 - \log 32}$
$= \log 4^3 - \log 32$
$= \log \frac{4^3}{32}$

(iii). $\mathbf{\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3}$
$= \frac{1}{3}\log_5 2^3 + \frac{1}{3}\log_5 3^3 - \log_5 3$

$= \log \frac{x^7}{y^6}$ (Answer)

$= \log \frac{\cancel{64}^{\,2}}{\cancel{32}}$

$= \log 2$ (Answer)

$= \log_5 2^{\cancel{3}\times\frac{1}{\cancel{3}}} + \log_5 3^{\cancel{3}\times\frac{1}{\cancel{3}}} - \log_5 3$

$= \log_5 2 + \log_5 3 - \log_5 3$

$= \log_5 2$ (Answer)

Question # 8: Expand the following using laws of logarithms:

(i). $\log(xyz^6)$

$= \log x + \log y + \log z^6$

$= \log x + \log y + 6\log z$

(Answer)

(ii). $\log_3 \sqrt[6]{m^5n^3}$

$= \log_3 (m^5n^3)^{1/6}$

$= \frac{1}{6}(\log_3 m^5n^3)$

$= \frac{1}{6}(\log_3 m^5 + \log_3 n^3)$

$= \frac{1}{6}(5\log_3 m + 3\log_3 n)$

(Answer)

(iii). $\log\sqrt{8x^3}$

$= \log(2^3x^3)^{1/2}$

$= \log(2x)^{3/2}$

$= \frac{3}{2}\log 2x$

$= \frac{3}{2}(\log 2 + \log x)$ (Answer)

Question # 9: Find the values of the following with the help of logarithm table:

(i). $\sqrt[3]{68.24}$

Let,

$x = \log(68.24)^{1/3}$

Taking 'log' on both sides

$logx = log(68.24)^{1/3}$

$logx = \frac{1}{3}log68.24$

$logx = \frac{1}{3}(1.8340)$

$logx = 0.6113$

Taking '$antilog$' on both sides

$\cancel{antilog} \times \cancel{log}x = antilog0.6113$

$x = 4.086$ (Answer)

(ii). 319.8×3.543

Let,

$x = 319.8 \times 3.543$

Taking 'log' on both sides

$logx = \log(319.8 \times 3.543)$

$logx = log319.8 + log3.543$

$logx = 2.5049 + 0.5494$

$logx = 3.0543$

Taking '$antilog$' on both sides

$\cancel{antiog} \times \cancel{log}x = antilog3.0543$

$x = 1133.182$ (Answer)

(iii). $\frac{36.12 \times 750.9}{113.2 \times 9.98}$

Let,

$x = \frac{36.12 \times 750.9}{113.2 \times 9.98}$

Taking 'log' on both sides

$logx = log\left[\frac{36.12 \times 750.9}{113.2 \times 9.98}\right]$

$logx = log36.12 + log750.9 - log113.2 - log9.98$

$logx = 1.5577 + 2.8756 - 2.0538 - 0.9991$

$logx = 1.3804$

Taking '$antilog$' on both sides

$\cancel{antilog} \times \cancel{log}x = antilog1.3804$

$x = 24.01$ (Answer)

Question # 10: In the year 2016, the population of a city was 22 million and was growing at a rate of 2.5% per year. The function $p(t) = 22(1.025)^t$ gives the population in million, t years after 2016. Use the model to determine in which year the population will reach 35 million. Round the answer to the nearest year.

$$\text{Population} = p(t) = 35\ millions$$
$$\text{Time} = t\ (Years) = ?$$
$$\because p(t) = 22(1.025)^t$$
$$35 = 22(1.025)^t$$
taking 'log' on both sides
$$log35 = log[22(1.025)^t]$$
$$log35 = log22 + tlog1.025$$
$$1.5441 = 1.3424 + t(0.0107)$$
$$1.5441 - 1.3424 = t(0.0107)$$
$$\frac{0.2017}{0.0107} = t$$
$$t = 18.85 \approx 19\ years$$
$$\text{Year when population will be 35 million} = 2016 + 19$$
$$= 2035$$