Admission Test - 27 March 2011

1. Prove that for any $n \in \mathbb{Z}$ the numbers n^2+4n+2 and $2n^2+9n+5$ are relatively prime.

2. a) In how many ways can be painted a row of 2011 houses, using four colors, if any two neighboring houses have different colors?

b) In how many ways can be painted these houses if any four consecutive houses have four different colors?

3. a) Find the maximum area of a triangle ABC for which

$$AB = BC = 1.$$

b) Find the maximum area of a quadrilateral $ABCD$ for which

$$AB = BC = CD = 1.$$

4. Let P denote the cartesian product of all finite cyclic groups

$$P = \Pi_{n\geq 1}\mathbb{Z}_n.$$

a) Show that the subset

$$S = \{(x_1, x_2, ...) \in P \mid x_k \neq \widehat{0} \text{ for only finitely many } k \in \mathbb{N}\}$$

is a subgroup of P.

b) Are the two groups S and P isomorphic?

5. Let $(\alpha_n)_{n\geq 1}$ and $(t_n)_{n\geq 1}$ be two sequences of real numbers, $t_n \geq 0$ for any n, and c a constant such that

$$\sum_{n\geq 1} t_n |\sin(nx + \alpha_n)| < c \text{ for any } x \in [0, \pi/2].$$

a) Show that $\sum_{n\geq 1} t_{2n+1} < \infty$.

b) Show that $\sum_{n\geq 1} t_n < \infty$.

Give a proof for all your statements.
Time allowed: 4 hours.
Each question is worth 20 points.
Do not use your calculator or mobile or other tables.
Organized by Abdus Salam School of Mathematical Sciences, GCU-Lahore

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