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# Admission Test Ph.D. 2008

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## Instructions

a. Maximum time allowed: 3½ hours.
b. Each problem worth 20 points.
c. Write only solutions of the problems, solution of each problem must start on separate page.
d. Give rigorous proofs for all your answers.

## PROBLEMS

**Problem 1:** *a*) Find the number of matrices $A \in M_2\{0,1\}$ which satisfy the condition $A^2 = A$.

*b*) Show that the number of matrices $A \in M_n\{0,1\}$ which satisfy the condition $A^2 = A$ is even.

**Problem 2:** We are given the fields $\mathbb{Q}(i) = \left\{a + bi \mid a,b \in \mathbb{Q}\right\}$ and $\mathbb{Q}\left(\sqrt{2}\right) = \left\{a + b\sqrt{2} \mid a,b \in \mathbb{Q}\right\}$.

*a*) Show that $\mathbb{Q}(i)$ and $\mathbb{Q}\left(\sqrt{2}\right)$ are isomorphic as $\mathbb{Q}-$vector spaces.

*b*) Show that $\mathbb{Q}(i)$ and $\mathbb{Q}\left(\sqrt{2}\right)$ are not isomorphic as fields.

**Problem 3:** Let $C\left(\mathbb{R}\right)$ be the space of continuous functions of one real variable and let

$$A : C\left(\mathbb{R}\right) \to C\left(\mathbb{R}\right)$$

be the operator which maps the function $f$ into the function $A(f)$ defined by the formula $\left(Af\right)(x) = 1 + \int_0^x f(t)dt$. Find the limit of the sequence of functions $f, A(f), A^2(f), A^3(f), \ldots$ when $f \equiv 1$.

**Problem 4:** Let $\Delta$ be the plane domain consisting of interior and boundary points of a rectangle $ABCD$ of sides $AB = a$ and $BC = b$. For every point $P \in \Delta$ one defines

$$f\left(P\right) = PA + PB + PC + PD,$$

where $PA$ means the length of the segment with endpoints $P$ and $A$. Find the range of the function $f$.

**Problem 5:** Let $E$ be the nonempty subset of the set $\left(0,+\infty\right)$ which satisfies the conditions:

(i) for any $x \in E \;\Rightarrow\; \frac{x}{2} \in E$,

(ii) for any $x, y \in E \;\Rightarrow\; \sqrt{x^2+y^2} \in E$ .

Prove that $\overline{E} = \left[0,+\infty\right)$.

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