Admission Test Ph.D. 2008

Merging man and maths

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Instructions

- a. Maximum time allowed: $3\frac{1}{2}$ hours.
- b. Each problem worth 20 points.
- c. Write only solutions of the problems, solution of each problem must start on separate page.
- d. Give rigorous proofs for all your answers.

PROBLEMS

Problem 1: *a*) Find the number of matrices $A \in M_2\{0,1\}$ which satisfy the condition $A^2 = A$.

b) Show that the number of matrices $A \in M_n\{0,1\}$ which satisfy the condition $A^2 = A$ is even.

Problem 2: We are given the fields $\mathbb{Q}(i) = \{a + bi \mid a, b \in \mathbb{Q}\}$ and $\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}.$

- a) Show that $\mathbb{Q}(i)$ and $\mathbb{Q}(\sqrt{2})$ are isomorphic as \mathbb{Q} vector spaces.
- b) Show that $\mathbb{Q}(i)$ and $\mathbb{Q}(\sqrt{2})$ are not isomorphic as fields.

Problem 3: Let $C(\mathbb{R})$ be the space of continuous functions of one real variable and let

$$A: C(\mathbb{R}) \to C(\mathbb{R})$$

be the operator which maps the function f into the function A(f) defined by

the formula $(Af)(x) = 1 + \int_0^x f(t)dt$. Find the limit of the sequence of functions $f, A(f), A^2(f), A^3(f), \dots$ when $f \equiv 1$.

Problem 4: Let Δ be the plane domain consisting of interior and boundary points of a rectangle *ABCD* of sides *AB* = *a* and *BC* = *b*. For every point $P \in \Delta$ one defines

$$f(P) = PA + PB + PC + PD,$$

where PA means the length of the segment with endpoints P and A. Find the range of the function f.

Problem 5: Let *E* be the nonempty subset of the set $(0, +\infty)$ which satisfies the conditions:

- (i) for any $x \in E \implies \frac{x}{2} \in E$,
- (ii) for any $x, y \in E \implies \sqrt{x^2 + y^2} \in E$.

Prove that $\overline{E} = [0, +\infty)$.

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