Admission Test Ph.D. 2007

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Instructions

Maximum time allowed: 3½ hours. Each problem worth 20 points. Write only solutions of the problems, each problem on separate sheet(s). Give rigorous proofs for all your answers.

PROBLEMS

Problem 1: For every positive integer n, consider the open intervals $\Delta_{n,k} = \left(\frac{k}{2^n}, \frac{k+1}{2^n}\right)$, where $k = 0, 1, \dots, 2^n - 1$. Define the function

$$r_n\left(x\right) = \begin{cases} (-1)^k & \text{if } x \in \Delta_{n,k} \\ 0 & \text{if } x = \frac{k}{2^n} \end{cases}$$

Give positive integers $n_1 \leq n_2 \leq \ldots \leq n_n$, Compute the integral

$$I(n_1,...,n_p) = \int_0^1 r_{n_1}(x) r_{n_2}(x) \dots r_{n_p}(x) dx$$

Problem 2: *a*) Show that the additive group $\mathbb{Z} \times \mathbb{Z}$ is not cyclic and find the minimal set of generators for it.

b) Let n > 0 be an integer, \mathbb{R} be the additive group of real numbers and U be the multiplicative group of complex numbers of modulo 1. Show that the group $\mathbb{R}/n\mathbb{Z}$

and U are isomorphic.

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Problem 3: A circle with center at O is divided into n equal arcs, $n \ge 2$, by the points $A_1, A_2, ..., A_n$. Find the sum of radius vectors $\overrightarrow{OA_i}$, i = 1, 2, ..., n.

Problem 4: Let *V* be the real vector space of the polynomials $P(X) \in \mathbb{R}[X]$ of degree at most 2 and $f: V \to V$ the function defined by f(P(X)) = P(X-1), for all $P(X) \in V$.

- a) Show that f is an isomorphism of vector spaces.
- b) Find the polynomial P(X) such that $f(P(X)) = X^2 + X + 1$.
- c) Find the matrix A of f with respect to the basis $\{1, X, X^2\}$.
- d) Find the invariant subspaces of f.
- e) Compute A^n , for all $n \ge 2$.

Problem 5: For any positive integer n let $C^n = [0,1]^n$ be the n-dimensional unite cube. Define

$$I_{n} = \int_{C^{n}} \min \left\{ x_{1}, x_{2}, \dots, x_{n} \right\} dx_{1} dx_{2} \dots dx_{n} \,.$$

- a) Compute I_2 and I_3 .
- b) Compute I_n for all $n \ge 2$.

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