Admission Test Ph.D. 2006

SCHOOL OF MATHEMATICAL SCIENCES, LAHORE

Time Duration: 3 Hours, Available at http://www.mathcity.org

Note: Attempt all questions.

Q 1. Let $P\left(\{0,1\}\right)$ denote the power set of the set $\{0,1\}$, We say that a function $f: P\left(\{0,1\}\right) \to \{1,2,\ldots,5\}$ is increasing function if for any $A,B \in P\left(\{0,1\}\right)$ we have $f(A) < f(B)$ whenever A is a proper subset of B. How many such increasing functions are injective and how many are not.

Q 2. Define $(a_n)_{n\geq 1}$ recursively as follows:

$$a_1 = 2 \quad , \qquad a_{n+1} = \frac{1}{2}\left(a_n + \frac{1}{a_n}\right) \text{ for } n \geq 1$$

Prove that sequence is convergent and find $\lim\limits_{n\to\infty} a_n$.

Q 3. If the polynomial $P \in \mathbb{R}[X]$ of degree $n \geq 2$ has all roots in $\mathbb{R}$, then all the roots of its derivative P' are also real numbers.

Q 4. Evaluate integral $\iint\limits_S xy\left(x^2+y^2\right)dx\,dy$, where S is the domain in the first quadrant bounded by the curves

$$x^2 - y^2 = 1 \;\; , \;\; x^2 - y^2 = 4 \;\; , \;\; xy = 1 \; , \; xy = 2$$

Q 5. (a) Show that for any bijective function $f:[n]\to[n]$, there exist positive integer k such that $f^k = id_{[n]}$, where f^k denotes composed function $f \circ f \circ f \ldots$ (k times).

(b) Find a bijection $f:\mathbb{N}\to\mathbb{N}$ such that $f^k \neq id_{\mathbb{N}}$ for any $k > 0$.

Unofficial Note:

The test was held on 04 May 2006 at School of Mathematical Sciences (SMS), 68-B, New Muslim Town, Lahore 54600. Pakistan.

And SMS announced that Admission Test will be a 3 hours long written exam and it will includes problems from the areas of Real Analysis, Linear Algebra, Abstract Algebra, Geometry, Combinatorics and Complex Analysis. These problems vary from B.Sc to M.Sc. level. (For more details visit http://www.sms.edu.pk)