Admission Test Ph.D. 2005

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Time Duration: 4 Hours, Available online at http://www.mathcity.org

Note: Choose the correct best answer(s) as the case may be.

1) Find: $\lim\limits_{n\to\infty} 8n^2\pi\sqrt{n^2+n+1}$

(a) 1 (b) -1 (c) 0
(d) 2 (e) π (f) ∞

2) Let $f:\mathbb{R}\to\mathbb{R}$, $f(x)=a|x+1|+|x-1|+(2-a)x-a-1$ and M be the set of values of $a\in\mathbb{R}$ for which f is bijective then

(a) $M=\phi$ (b) $M=\mathbb{R}$ (c) $M=(1,\infty)$
(d) $M=(-\infty,0)$ (e) $M=(-1,1)$ (f) $M=\left(-\infty,\frac{1}{2}\right)$

3) Let $f:(-\infty,-1]\bigcup\left[-\frac{1}{4},\infty\right)\to\mathbb{R}$, $f(x)=x\sqrt{\dfrac{x+1}{4x+1}}$, then

(a) $y=x+1$ is obviously asymptote to ∞, f is concave.
(b) f is decreasing.
(c) f is convex.
(d) $x=0$ is vertical asymptote.

4) $f:\mathbb{R}\to\mathbb{R}$ be such that $\left|f(x)-x^3\right|\le 2x^2\ \ \forall\ \ x\in\mathbb{R}$, then

(a) f is continuous at $x=0$
(b) f admits an obligue asymptote.
(c) $f'(0)$ exists and $f'(0)\neq 1$
(d) f admits a horizontal asymptote.
(e) We can decide differentiability of f at $x=0$.
(f) $f'(0)$ exists and $f'(0)=0$.

5) Find $m>0$ such that the area of the set

$$M=\left\{(x,y)\in\mathbb{R}^2 \mid m\le x\le 2m, 0\le y\le x+\tfrac{6}{x^2}\right\}$$

(a) 2 (b) 3 (c) $\dfrac{1}{2}$
(d) 1 (e) 4 (f) 10

6) Let $A \in \mathrm{M}_2(\mathbb{R})$ such that $A^3 = -I_2$. Find maximum numbers of distinct matrices of the form A^n with $n \geq 1$.

(a) 8 (b) ∞ (c) 1
(d) 12 (e) 10 (f) 6

7) Find: $$\lim_{a\to 1}\lim_{x\to\infty}\left(\frac{1+a+a^2+...+a^n}{1+a^2+a^4+...+a^{2n}}\right), \quad a<1$$

(a) 0 (b) ∞ (c) 2
(d) 1 (e) 3 (f) Does not exists.

8) $\lim f(x) = \int_0^a \frac{x^p}{\sqrt{x^3+a^2}}$; $a>0, p>0$ then f is constant if

(a) $p=1$ (b) $p=\frac{1}{2}$ (c) $p=0$
(d) $p=2$ (e) $p=\frac{3}{2}$ (f) $p=\frac{5}{2}$

9) Let $a_n = 1+2\cdot 2+3\cdot 2^2+...+n\cdot 2^{n-1}$, $n\in\mathbb{N}$ at $l=\lim_{n\to\infty}\frac{a_n}{n\cdot 2^n}$, then

(a) $l=1$ (b) $l=3$ (c) $l=\infty$
(d) $l=2$ (e) $l=0$ (f) $l=\frac{1}{2}$

10) Let $A \in \mathrm{M}_2(\mathbb{R})$ and n is the number of solutions of equation $AX - XA = I_2$, then

(a) $n=1$ (b) $n=0$ (c) $n=2$
(d) $n=3$ (e) $n=4$ (f) $n=5$

11) Let **M** be the set of the set of polynomials of degree two or less than two is an abelian group addition of polynomials. Let $m\in\mathbb{Z}$ such that $f_m : \mathbf{M}\to\mathbf{M}$, $f_m(p) = x^2p'' + (x+3)p' + mp \;\; \forall \;\; p\in\mathbf{M}$, is an isomorphic then

(a) m is any integer (b) $m\neq 0,4$ (c) $m\neq 0,-4,-1$
(d) $m\neq 1$ (e) $m\neq 0$ (f) $m\neq 0,4,-2$

12) Consider a sphere with centre at point P and radius R. A point source of light is situated at the distance $2R$ from P. Find area of eliminated part.

(a) $\frac{\pi R^2}{3}$ (b) R^2 (c) $\frac{3\pi R^2}{4}$
(d) $\frac{2\pi R^2}{4}$ (e) $3R^2$ (f) πR^2

13) Find $a \in \mathbb{R}$ such that $x^2 + y^2 - 3x - y + a > 0 \quad \forall \; x, y \in \mathbb{R}$.

(a) $a \in (1, \infty)$ (b) $a \in (-1, 1)$ (c) $a = 7$

(d) $a \in \left(\frac{5}{2}, \infty\right)$ (e) $a \in (-\infty, -1)$ (f) Doesn't exist

14) Let the sequence of functions $(f_n, n \geq 1)$, $f_n : \mathbb{R} \to \mathbb{R}$ be defined as

$f_1(x) = \sqrt{x}$, $f_2(x) = \sqrt{x + \sqrt{x}}$, … , $f_n(x) = \sqrt{x + \sqrt{x + \sqrt{x + \ldots}}}$

Take $f : D \to \mathbb{R}$

(a) $\lim\limits_{n \to \infty} f_n(x) = f$, where D is maximum domain of f then $D = (0, \infty)$

(b) f is not continuous on D (c) f is not differentiable at $x = 0$

(d) $D = [1, \infty)$ (e) f is increasing

(f) $D = \left[-\dfrac{1}{4}, \infty\right)$

15) Calculate: $\dfrac{x_1^3 + x_2^3 + x_3^3}{x_1^2 + x_2^2 + x_3^2}$, where x_1, x_2 and x_3 are zeros of $x^3 - 3x + 1$

(a) 0 (b) $-\dfrac{1}{2}$ (c) 2

(d) -2 (e) 10 (f) $2 + i$

16) - *Missing*

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