Written Test Lecturer 2004

PUNJAB PUBLIC SERVICE COMMISSION, LAHORE.

Mathematics , Available online @ http://www.mathcity.org

Time Allowed: 3 hours **Maximum Marks: 100**

NOTE: Attempt any five questions.

Q 1: (a) Solve the differential equation:

$$\frac{d^2y}{dx^2} + 4y = 4\tan 2x \qquad (10)$$

(b) Prove that:

$$\frac{\sin 5\theta}{\sin\theta} = 16\cos^4\theta - 12\cos^2\theta + 1 \qquad (10)$$

Q 2: (a) The greatest resultant that forces can have is of magnitude P and the least is of magnitude Q. Show that when they act at an angle α, their resultant is of magnitude

$$\sqrt{P^2\cos^2\frac{\alpha}{2} + Q^2\sin^2\frac{\alpha}{2}}. \qquad (10)$$

(b) Prove that

$$(\mathbf{a}\times\mathbf{b})(\mathbf{c}\times\mathbf{d}) + (\mathbf{c}\times\mathbf{a})(\mathbf{b}\times\mathbf{d}) + (\mathbf{b}\times\mathbf{c})(\mathbf{a}\times\mathbf{d}) = 0 \qquad (10)$$

Q 3: (a) Prove that any metric space is Hausdorff space. (8)

(b) Let $(\mathbf{X}, \tau_{\mathbf{X}})$ and $(\mathbf{Y}, \tau_{\mathbf{Y}})$ be topological spaces, $f : \mathbf{X} \to \mathbf{Y}$ be a mapping, $\mathbf{A} \subseteq \mathbf{X}$. Prove that f is continuous if and only if $f(\overline{\mathbf{A}}) \subseteq \overline{f(\mathbf{A})}$, where $\overline{\mathbf{A}}$ is closure of $\mathbf{A}$. (12)

Q 4: (a) Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be a linear mapping defined as

$$T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_1 - x_3, x_1)$$

Find (*i*) Matrix of T w.r.t standard basis (*ii*) $\operatorname{Ker} T$ (*iii*) $\operatorname{Im} T$ (12)

(b) Show that $\{(1,1,1),(0,1,1),(0,0,1)\}$ is basis of $\mathbb{R}^3$. Using Gram-Schmidt Orthogonalization Process, transform this basis into orthonormal basis. (8)

Q 5: (a) Let $\mathbb{R}$ be the set of real numbers, $\mathbf{M}_3(\mathbb{R})$ be the set of all 3×3 matrices over $\mathbb{R}$ and let $GL_3(\mathbb{R}) = \{A \in \mathbf{M}_3(\mathbb{R}) : \det A \neq 0\}$. (10)

(b) Prove the following claims

(*i*) $\mathbf{M}_3(\mathbb{R})$ is a vector space of dimension 9.

(*ii*) $GL_3(\mathbb{R})$ forms a non abelian group with centre consisting of non-zero scalar matrices. (10)

Q 6: (a) A farmer has 50 ton of cattle which he can sell at a profit of Rs. 120 per ton. If the cattle gain weight by 5 ton per week, but the profit falls by Rs. 4 per ton, when should he sell the cattle to obtain the maximum profit? (10)

(b) Find the shortest distance between the lines

$$x + a = 2y = -12z \quad \text{and} \quad x = y + 2a = 6z - 6a \qquad (10)$$

Q 7: (a) Prove that a monotonic increasing sequence which is bounded above, converges to its least upper bound. (10)

(b) Discuss the existence of the Riemann integration over a closed interval $[a, b]$. (10)