Written Test Lecturer 2001

PUNJAB PUBLIC SERVICE COMMISSION, LAHORE.

Time Allowed: 3 hours **Maximum Marks: 100**

NOTE: Attempt any five questions.

Q 1: (a) If G is an abelian group, show that

$$(ab)^n = a^n b^n \quad \text{for all } a,b \in G \text{ and } n \in \mathbb{Z}. \qquad (10)$$

(b) Let $(R,+,\cdot)$ be a ring such that $a^2 = a$ for all $a \in R$. Prove that

(i) $2a = 0$ for all $a \in R$ (ii) $ab = ba$ for all $a,b \in R$ (10)

Q 2: (a) Show that the system of equations

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= a \\ 3x_1 + x_2 - 5x_3 &= b \\ -5x_1 - 5x_2 + 21x_3 &= c \end{aligned}$$

is inconsistence if $c \neq 2a - 3b$. (10)

(b) Find a basis and dimension of the subspace W of $\mathbb{R}^4$ spanned by

$(1,4,-1,3)$, $(2,1,-3,-1)$ and $(0,2,1,-5)$ (10)

Q 3: (a) A 6 feet tall man is walking towards a lamp post 16 feet high at a speed of 5 feet/sec (i) At what rate is the tip of his shadow moving? (ii) At what rate is the length of his shadow changing? (10)

(b) If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, showing working clearly, prove that

$$\frac{d^2y}{dx^2} = \frac{abc + 2fgh - af^2 - bg^2 - ch^2}{(hx + by + f)^3} \qquad (10)$$

Q 4: (a) If the forces $p\overrightarrow{AB}$, $q\overrightarrow{BC}$, $r\overrightarrow{CD}$ and $s\overrightarrow{AD}$ acting along the sides of a plane quadrilateral are in equilibrium, show that $pr = qs$. (10)

(b) Determine the position of the centroid of the area in the first quadrant enclosed by the circle $x^2 + y^2 = a^2$. Determine also the centroid of the arc length. (10)

Q 5: (a) Prove that (i) $\operatorname{curl}\operatorname{grad} f = 0$ and (ii) $\operatorname{div}\operatorname{curl} f = 0$ (10)

(b) Find the length and equation of the shortest distance between the straight line joining the points $A(3,2,-4)$ and $B(1,6,-6)$ and the straight line joining the points $C(-1,1,-2)$ and $D(-3,1,-6)$. (10)

Q 6: (a) Compute $\mathcal{L}\left\{te^{at}\cos bt\right\}$, (where $\mathcal{L}$ denote the Laplace transformation.) (10)

(b) Prove that a convergent sequence in a metric space is bounded and its limit is unique.

Q 7: (a) Prove that

$$\sin\theta + \sin(\theta+\alpha) + \sin(\theta+2\alpha) + \ldots + \sin(\theta+n\alpha) = \frac{\sin(n+1)\dfrac{\alpha}{2}}{\sin\dfrac{\alpha}{2}}\sin\left(\theta + n\frac{\alpha}{2}\right) \qquad (10)$$

(b) Find the point on the straight line $2x - 7y + 5 = 0$ that is closest to the origin. (10)