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Note: Attempt any five questions, all carry equal marks.

Question 1(a): Give an example of a relation on a set:

(i) Which is reflexive, symmetric but not transitive.

(ii) Which is symmetric and transitive but not reflexive.

(iii) Which is reflexive, transitive but not anti-symmetric.

(b): Let $f : X \leftarrow Y$ be a function and A, B be subsets of X. Show that

(i) $f(A \cup B = f(A) \cup f(B)$.

(ii) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$

Question 2(a): Show that the set F of all those elements of an abelian group A which have finite order is a subgroup of A.
(b): Let G be a group and $G^2 = \{(x, y) : x, y \in G\}$. Show that G^2 is a group under the multiplication defined by:
$(x, y)(x_1, y_1) = (xx_1, yy_1)$ for all $x, x_1, y, y_1 \in G$. Also show that G is embedded in G^2.

Question 3(a): Show that the complex valued functions given by:

$$f_1(z) = z, f_2(z) = \frac{1}{1-z}, f_3(z) = \frac{z-1}{z}, f_4(z) = \frac{1}{z}, f_5(z) = 1 - z, f_6(z) = \frac{z}{z-1}$$

form a group under the usual multiplication of mappings. Is this group abelian? Justify.
(b): Define an ideal of a ring. Write three ideals of the ring $\mathbb{Z}$ of integers. Are they maximal?

Question 4(a): Using the properties of the determinants prove that:

$$\begin{pmatrix} 2 & 4 & 1 & 12 \\ -1 & 1 & 0 & 3 \\ 0 & -1 & 0 & -3 \\ 7 & 3 & 6 & 9 \end{pmatrix} = 0$$

(b): Let

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 1 & 3 \\ 0 & -2 & 5 \end{pmatrix}, B = \begin{pmatrix} -3 & 2 & 4 \\ 4 & 0 & 6 \\ 5 & -2 & 3 \end{pmatrix}$$

Find $A \cdot B, B \cdot A$ and show, by evaluating $A \cdot B$, that $A \cdot B = B \cdot A$.

Question 5(a): The correspondence between measurements of temperatures in degree Fahrenheit and degree Centigrade is given by the formula:

$$5F - 9C = 180$$

where F is on the Fahrenheit scale and C is on the Centigrade scale. Express C as a linear function of F. Find its graph and determine its slope. Also find corresponding measurements for the given measurements.
(i) $30°F$ (ii) $212°F$ (iii) $50°C$.
(b): Show that among the rectangles having a fixed perimeter P, the square has the maximum area.

Question 6(a): Explain the following concepts with an example of each.
(i) Sequence (ii) Bounded sequence (iii) Convergent sequence (iv) Range of the sequence.
(b): Examine the convergence of the following series:

$$(i) \quad \frac{1}{n(\log n)^p} \quad (ii) \quad \frac{(n+1)^{-n}}{n}.$$

Question 7(a): Let

$$f(x) = \begin{cases} \frac{x^2}{2} - 2, & \text{if } 0 < x < 2; \\ 2 - \frac{8}{x^2}, & \text{if } x > 2. \end{cases}$$

Does f have limit at $x = 2$? If it does then find the limit.
(b): Discuss the continuity of the following functions.
(i) $f(x) = x$. (ii) $g(x) = 5$. (iii) $h(x) = [x]$.
for all $x \in \mathbb{R}$. Are these differentiable.

Question 8(a): Justify your answer to the following:

(a) Is the set $\mathbb{Z}$ of integers closed in the real line $\mathbb{R}$.

(b) Is every metric space a T_1 space.

(c) Is an infinite set with cofinite topology a Hausdorff space.

(d) Is the intersection of any two closed sets is a metric space is closed.