

QUAID-I-AZAM UNIVERSITY  
DEPARTMENT OF MATHEMATICS  
M.Phil. Admission Test Spring 2014

Time: 90 minutes

Dated: 22-01-2014

Notes: Use separate page for each question.

Q.1. Show that the sequence  $(y_n)$ , where  $y_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \frac{1}{(n+3)^2} + \dots + \frac{1}{(n+n)^2}$

converges to zero.

Q.2. Let  $V$  be an  $n$  dimensional vector space over a field  $F$ . Show that  $L(V, V) \cong M(n, F)$ .

Q.3. Let  $r(s)$  be a unit speed curve. Then for every  $s$  such that  $\kappa(s) \neq 0$ , show that the set  $\{T(s), N(s), B(s)\}$  is an orthonormal set.

Q.4. Let  $\mathbb{R}$  be the set of all real numbers and  $\mathfrak{T}$  consists of  $\mathbb{R}$  and all those subsets of  $\mathbb{R}$  which do not contain 2. Prove or disprove that  $\mathfrak{T}$  is a topology.

Q.5. Solve  $y'' + (\cos x)y = 0$ .

Q.6. Show that any two circles in the plane with the same center are Bertrand curves.

Q.7. Reduce the following into canonical form and hence solve it  $u_{xx} - 2 \sin x u_{xy} - \cos^2 x u_{yy} - \cos u_y = 0$ .

Q.8. A particle moves in a plane with constant speed. Prove that its acceleration is perpendicular to its velocity.

Q.9. Give an example of a linear operator which is unbounded.

Q.10. Use Chebyshev method, find the root of the equation  $\cos x - xe^x = 0$ .

Q.11. Show that  $u(x, y)$  is harmonic in some domain and find a harmonic conjugate  $v(x, y)$  when  $u(x, y) = \sinh x \sin y$ .

Q.12. If  $G$  is a non-abelian group of order 6 then prove that  $G \cong S_3$ .

Q.13. Examine the convergence of  $\int_0^{\infty} \cos x^3 dx$ .

GOOD LUCK