QUAID-I-AZAM UNIVERSITY
DEPARTMENT OF MATHEMATICS
M. Phil. Admission Test Spring 2013

Time: 90 minutes

Dated: 21-01-2013

Note: Use separate page for each question.

Q.1. Discuss the continuity of $f(x,y)$ at the origin. where

$$f(x,y) = \begin{cases} \frac{x^2 y^2}{x^3 + y^3} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Q.2. Find $T(x,y)$, where $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that $T(1,2) = (3,1,-5)$ and $T(0,1) = (2,1,-1)$. Also find the kernel of T.

Q.3. Let X be a non-empty and $\Im$ be the co-finite topology on X. Show that X is a compact space.

Q.4. Compute curvature and torsion of the curve

$$\alpha(t) = (\cos t, \sin t, t)$$

Q.5. Solve the initial value problem

$$y'' - 4y' + 3y = 10e^{-2x}, \quad y(0) = 1, y'(0) = -3$$

Q.6. Show that if H is a cyclic normal subgroup of a group G. then every subgroup of H is normal in G.

Q.7. Find the moment of inertia of a rod of mass m and length $2a$ about a line passing through the centre of the rod and perpendicular to it.

Q.8. Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$, where c is a circle $|z| = 3$.

Q.9. Test the convergence of the integral

$$\int_1^\infty \frac{\cos x + \sin 3x}{x^{\frac{5}{4}}} dx$$

Q.10. Find the solution of

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k} \frac{\partial u}{\partial t}, \quad 0 < x < 1, t > 0$$

$$u(0,t) = 1, \quad u(1,t) = 1, \quad t > 0$$

$$u(x,0) = 1 + \sin \pi x \quad 0 < x < 1$$

Q.11. Let X be an inner product space and $u, v \in X$. If $(x, u) = (x, v)$ for all $x \in X$, then $u = v$.

Q.12. Evaluate $\int_0^{90°} \cos x \, dx$ by using Simpson's $\frac{1}{3}$ rule for $n = 6$.

1