M.Phil Admission Test Spring 2008

QUAID-E-AZAM UNIVERSITY, ISLAMABAD.

Mathematics, Dated: 4-2-2008, http://www.mathcity.org

Time: **9:30 a.m. to 11:00 a.m.**
Dated: **4-02-2008.**
Note: **Section I is Compulsory, Section II is for Applied Mathematics Candidates and Section III is for Pure Mathematics Candidates.**

Section I

Q.1. Evaluate the following integral

$$\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)dy\, dx.$$

Q.2. Let $\beta(s)$ be a unit speed curve in $\mathbb{R}^3$ such that $\kappa \neq 0$ is a constant and $\tau = 0$. Then Prove that $\beta(s)$ is a circle of radius $\frac{1}{\kappa}$.

Q.3. Define moment of a force. Find at any time t the moment of the force $\underline{F} = t\underline{i} + t^2\underline{j} + (2t^3-1)\underline{k}$ acting on a particle of mass m whose position vector relative to an origin O is given by $\underline{r} = 2\underline{i} + 3\underline{j} - 5\underline{k}$.

Q.4. Find the contracted tensor components of G^a_b, where $G^{ac}_{db} = A^a_d B^c_b$ and $A^a_d = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}$, $B^c_b = \begin{bmatrix} -2 & -1 \\ 4 & 3 \end{bmatrix}$.

Q.5. a) Determine residue of $f(z) = \frac{1}{z(e^z-1)}$.

b) Determine the number of zeros, counting multiplicities of the polynomial $2z^5 - 6z^2 + z + 1$ in the annulus $1 \leq |z| \leq 2$.

Q.6. Let G be a group and $N \leq G$, $K \leq G$ such that $N \vartriangle K$. Then Prove that $K/N \vartriangle G/N$.

Q.7. Show that in a normed space every convergent sequence is bounded. Give an example to show that converse is not always true.

Q.8. a) Verify that $\mathbb{Q} \oplus \mathbb{R}$ is a vector space over the field $\mathbb{Q}$ but not a vector space over $\mathbb{R}$.

b) What are the dimensions of quotient spaces $\frac{\mathbb{R}\oplus\mathbb{C}}{\mathbb{R}}$ and $\frac{\mathbb{C}^n}{\mathbb{R}}$.

c) Define algebra and division algebra.

Q.9. Let X be a non empty set and $\Im$ consists of empty set and all those subsets of X whose complement is finite. Show that $\Im$ is a topology on X.

Q.10. Evaluate $\int_0^{90^0} \cos x\, dx$ by using Simpson's $\frac{1}{3}$ rule for $n = 6$.

Q.11. Solve $y'' - 2y' + y = 10e^{-2x}\cos x$ by the method of undetermined coefficients — Annihilator approach.

Q.12. Show that the function $f(x)$ defined as follows

$$f(x) = \begin{cases} 1 & x \text{ is rational} \\ 0 & x \text{ is irrational} \end{cases}$$

is nowhere differentiable. What can you say about the integrability of this function.

Q.13. Solve

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, \quad t > 0$$
$$u(0,t) = 0, \quad t > 0,$$
$$u(x,0) = f(x), \quad 0 < x < \infty$$

and u, $\frac{\partial u}{\partial x}$, **both tend to zero as** $x \to \infty$.

Section II

Q.14. A velocity field is given by $u = 3x^2$, $v = 2x$, $w = 0$ **in arbitrary units. Is the flow is steady or unsteady? Is it One, two or three dimensional? At** $(x, y, 0) = (2, 1, 0)$ **compute the total acceleration.**

Q.15. Calculate the energy eigenvalues and eigenstates for a one dimensional harmonic oscillator.

Q.16. State the two basic postulates of the special theory of relativity. Use them to deduce the Lorentz transformation between coordinates (x, y, z, t) **and** (x', y', z', t') **for uniform relative velocity** v **along the** x**-direction.**

Q.17. The Lagrangian function describing the dynamics of a particle is given by $L = \frac{1}{2}(\dot{q}_1^2 + \dot{q}_2^2) - \frac{1}{2}(q_1^2 + q_2^2)$. **Find the Hamilton equations of motion for this system.**

Q.18. Find the resolvent kernel to solve the following equation

$$u(x) = f(x) + \lambda \int_0^x e^{x-t} u(t)\,dt.$$

Q.19. U Runge-Kutta method of fourth order to solve
$y' = xy + y^2$, $y(0) = 1$ **for** $y(0.1)$.

Q.20. Write the Maxwell's equations for electromagnetic medium and explain all relevant terms.

Section III

Q.21. Prove that a group of order 15 is not simple.

Q.22. Consider $X = \mathbb{R}^2$ **be an inner product space and let** $M = \{(1, 2)\}$. **Find** $M^{\perp}$

Q.23. Let R **be a commutative ring with identity and** I **be an ideal in** R. **Prove that** R/I **is an integral domain iff** I **is prime ideal.**

Q.24. Indicate true or false.

a) Every module is a vector space.

b) Every vector space is a module.

c) An additive abelian group is a module over $\mathbb{Z}$.

d) An additive abelian group is a module over $\mathbb{Q}$.

e) Every primary ideal of a commutative ring R with identity is prime ideal.

Q.25. Show that every free module is projective.

Q.26. Show that outer measure of an interval is its length.

Q.27. Show that if $P[a \leq x \leq b] = 1$ then $a \leq E[x] \leq b$.

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