

Time: 9:30 a.m. to 11:00 a.m.

Dated: 3-02-2007.

Note:- Section I is compulsory, section II is for Applied Mathematics and section III is for Pure Mathematics.

Section I

Q.1 Find a Fourier Series solution of

$$x'' + 4x = 4t$$

$$x(0) = x(1) = 0$$

Q.2 Solve the following

$$u_t(x,t) = u_{xx}(x,t) - h u(x,t) \quad 0 < x < \pi, t > 0$$

$$u(0,t) = 0 \quad t > 0$$

$$u(\pi,t) = 1 \quad t > 0$$

$$u(x,0) = 0 \quad t = 0$$

Q.3 Let  $X$  be an inner product space and  $u, v \in X$ . If  $\langle x, u \rangle = \langle x, v \rangle$  for all  $x \in X$  then  $u = v$ .

Q.4 Find the Gaussian and mean curvatures on  $\underline{x} = (u+v, u-v, uv)$  at  $u=1, v=1$ .

Q.5 Determine if the following integrals converges or diverges.

(i)  $\int_3^{\infty} \frac{dx}{x - e^{-x}}$

(ii)  $\int_1^{\infty} e^{-x^2} dx$ .

Q.6 Find the residue of  $f(z) = \frac{1}{z^2 \sinh z}$  at  $z=0$ .

Q.7 Let  $G$  be a finite group and  $H$  be a subgroup of  $G$ . Show that order of  $H$  divides the order of  $G$ .

(1)

Q.8 Obtain the geodesic equations for the metric

$$(ds)^2 = dr^2 + r^2 (d\theta)^2.$$

Q.9 a) Verify that  $\mathbb{R}$  is not a metric space whenever

$$d(x, y) = |\sin(x - y)|, \quad \text{where } x, y \in \mathbb{R}.$$

b) Define base and subbase for a topological space  $(X, \mathcal{T})$ . What will be the base and subbase for a discrete topological space.

Q.10 Let  $W$  be a subspace of the vector space  $V$  over the field  $F$ . Then  $V/W = \{v + W : v \in V\}$  is a vector space over  $F$ .

Q.11 Define moment of inertia for a continuous mass distribution. Find the moment of inertia of a uniform rod of mass  $m$  and length  $2a$  about a line perpendicular to the rod and lies at a distance  $\frac{2a}{3}$  from one end of the rod.

Q.12 The population of a certain town (as obtained from census data) is shown in the following table:

years	1965	1975	1985	1995	2005
Population in thousands	19.96	36.65	58.81	77.21	94.61

Find the rate of growth of the population in the year ~~1995~~ 1995.

Q.13 Show that the function

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

is continuous at the origin.

Section II      Applied Maths

Q. 14. Write the equation of motion for a viscous fluid when velocity is  $\vec{v} = (0, 0, u(y))$ .

Q. 15 Hooke's law for homogeneous isotropic elastic bodies is given by

$$\tau_{ij} = \lambda \delta_{ij} \epsilon_{kk} + 2\mu \epsilon_{ij}$$

Derive a relation between stress and strain components for

$$\tau_{ij} = \begin{bmatrix} T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Q. 16 Solve  $u(x) = x - \int_0^x (x-t) u(t) dt$ .

Q. 17 Define Pauli spin matrices. State and prove a commutation relation for Pauli spin matrices.

Q. 18, Test whether the motion specified by  $\vec{v} = \frac{k^2(x\hat{j} - y\hat{i})}{x^2 + y^2}$ ,

$k = \text{constant}$  is a possible motion for an incompressible fluid. If so determine the equations of the streamlines

Q. 19 ✓ The dynamical behaviour of a mechanical system is described by the hamiltonian function given by  $H = \frac{1}{2} (q_2 \dot{p}_1^2 + q_1 \dot{p}_2^2) + \frac{1}{2} (q_1^2 + q_2^2)$

Find hamilton's equation of motion.

Let  $\phi = p_1^2 + q_1^2 - q_1 q_2 + q_2^2$ . Is  $\phi$  an integral of hamilton's equation of motion or not?

Q. 20 Using modified Euler's method, obtain the solution of the differential equation at  $\frac{dy}{dx} = x + \sqrt{y}$  with the initial condition  $y(0) = 1$ , for the range  $0 \leq x \leq 0.6$  in steps of 0.2

Section III Pure Maths

Q.21 Let  $X, Y$  be independent, each with standard normal distribution. Find the distribution of

$$V = \frac{X}{Y}.$$

Q.22 Show that a sublinear functional  $p$  satisfies  $p(0) = 0$  and  $p(-x) \geq -p(x)$ .

Q.23 Show that every free  $R$ -module is projective.

Q.24 Show that every set of <sup>out</sup>measure zero is measurable.

Q.25 Find all Sylow 2 sub-groups of the symmetric group of degree 3, that is  $S_3$ .

Q.26 a) Define ring & ideal of a commutative ring with 1.

b) Let  $R$  be a commutative ring with identity and  $I$  be an ideal in  $R$ . Prove that  $R/I$  is an integral domain iff  $I$  is prime ideal. (1,4)

Q.27 a) Indicate true or false

- (i) Every additive abelian group is  $\mathbb{Z}$ -module.
- (ii)  $\mathbb{Z}$  is  $\mathbb{Z}$ -module.
- (iii)  $\mathbb{Z}$  is not  $\mathbb{Q}$ -module.
- (iv) Every vector space is free module.
- (v)  $\mathbb{Z}$  is free module over  $\mathbb{Z}$ .

b) verify that  $M_2(\mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R} \right\}$  is vector space over  $\mathbb{R}$ .

(2.5, 2.5)

Good Luck

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