M.Phil Admission Test Spring 2005

QUAID-E-AZAM UNIVERSITY, ISLAMABAD.

Mathematics, Dated: 26-1-2005, http://www.mathcity.org

~~Test~~ for admission to M.Phil. at Q.A. University for Spring semester 2005. Time: 9:30 – 11:00 Date 26/01/05

Note: Attempt as many questions as you can.

Q.1 Establish the following limit $\lim_{x \to c} \sqrt{x} = \sqrt{c}$ for any $c > 0$, using the $\epsilon - \delta$ definition of limit.

Q.2 (i) What are the dimensions of vector spaces $\mathbb{C}$ over real field and $\mathbb{C}$ over itself.
(ii) Define Algebra and give one example. (2, 8)

Q.3 Calculate the curvature and torsion of the curve.
$\underline{x} = (a \cos u, a \sin u, b u)$ a, b are constants

Q.4 Let $\mathbb{N}$ be the set of natural numbers and $E_n = \{1, 2, \ldots n\}$ for each $n \in \mathbb{N}$. Let $\mathcal{J}$ consists of empty set, $\mathbb{N}$ and E_n for each $n \in \mathbb{N}$. Show that $\mathcal{J}$ is a topology on $\mathbb{N}$.

Q.5 Solve the following differential equation by variation of parameter: $\dfrac{d^2y}{dx^2} + y = \sec x \cdot \tan x$

Q.6 Let C_R denote the upper half of the circle $|z| = R$ $(R > 2)$ taken in the counter clockwise direction. Without evaluating the integral, show that the value of the integral
$$\int_{C_R} \frac{2z^2 - 1}{z^4 + 5z^2 + 4} \, dz$$
tends to zero as R tends to infinity.

Q.7 Find the Fourier transform of $f(x) = e^{-a|x|}$.

Q.8 Let G be a group and H, K be normal subgroups in G. Define $\phi : G/H\cap K \rightarrow G/H \times G/K$ and prove that it is a monomorphism.

Q.9 Show that a Cauchy sequence of real numbers is bounded.

Q.10 (a) Give rank and value of the following tensors

(i) $R^{ab}_{\ \ cd} + \ln S^{ab}_{cd}$ (ii) $P^{ab}_{cd}\ Q^{d}_{e}$

(b) Are the following valid tensor expressions:

(i) $A^{ab}\ B_{bc} + \lambda\ R^{ae}\ Q_{ec}$

(ii) $Q^{ad}_{e}\ R^{e}_{g} + F^{ad}_{eg}$

(iii) $Q^{abc}_{d}\ P^{d}_{f} - S^{abf}_{d}\ E^{d}_{g}$

11 Find the upper bound for error term in the trapezoidal rule for the integral $I = \int_1^2 \left(\frac{e^{-x}}{x}\right) dx$.

12 Show that $C[a,b]$ is not complete under the norm defined by $\|f\| = \int_{-1}^{1} |f|\, ds$

base mechanics

13 Given the formula for the law of force in a central orbit as $F = h^2 u^2 \left[u + \frac{d^2u}{d\theta^2}\right]$

Find the law of force for the central orbit given by the equation $\frac{l}{r} = 1 + e\cos\theta$

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Find the constant of propartionaly as well.

Q.14 Calculate the smallest eigenvalue and corresponding eigenvector of the matrix

$$\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$ perform three iterations only.

Q.15 A car with six spark plugs is known to have two malfunctioning spark plugs. If two plugs are pulled at random, what is the probability of getting both of the malfunctioning plugs?

Q.16 Classify the following integral equation

$$u(x) = x - \int_0^x (x-t)\, u(t)\, dt$$

verify, which of the following is a solution of this equation $u(x) = \sin x$; $u(x) = x$

Q.17 Dynamical behaviour of a particle executing simple harmonic motion is described by the Lagrangian function

$$L = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} m \omega^2 x$$

Find Hamilton's equation of motion. Show that

$$Q = \frac{1}{2m} p_x^2 + \frac{1}{2} m \omega^2 x$$ is an integral of these equations of motion.

Q.18 Write down Maxwell's equations and give their mathematical interpretation.

Q.19 A velocity field is given by $\bar{V} = ax\,\hat{i} + ay\,\hat{j} + bxyt\,\hat{k}$ where $a = 2\ \sec^{-1}$, and $b = 1\ m^{-1}\sec^{-2}$. Determine the number of dimensions of the flow field. Is it steady? Find the slope of the stream line through $(1,2,0)$ at $t = 0$.

Q.20 The length of a moving rod appears to be decreased by one percent as seen by an observer at rest. Determine the speed of the rod.

Q.21 Prove that $|GL(n,q)| = (q-1)\,|SL(n,q)|$

Q.22 Let R be a commutative ring with identity and I be a ideal in R. Prove that R/I is integral domain if and only if I is prime ideal.

Q.23 Show that a divisible abelian group is injective as Z-module.

Q.24 Show that outer measure of an interval is its length.

Good Luck