QUAID-I-AZAM UNIVERSITY
DEPARTMENT OF MATHEMATICS
M.Phil .Admission Test Fall 2011

Time:9 0 minutes

Dated:2 2-08-2011

Note:Section Iis compulsory,Section II is for Applied Mathematics and Section III is for Pure Mathematics candidates.

Use separate page for each question.

Section I

Q.1 . (a) Investigate the limit at $(2,1)$ of $f(x,y) = \frac{\mathrm{Sin}^{-1}(xy-2)}{\tan^{-1}(3xy-6)}$.

(b) Evaluate $\int_c xy^2dx$,on the quarter circle C defined $x = 4\cos t, y = 4\sin t, 0 \le t \le \frac{\pi}{2}$.

Q.2 . Determine the radius of convergence for the power series $\sum_{1}^{\infty} \frac{(n!)^2}{(2n)!}x^n$.

Q.3 . Fill in the blanks

(a) If the radius of curvature of a curve is infinite then it is a ......

(b) If the radius of curvature of a curve is constant $\neq 0$ then it is a ......

(c) If the radius of curvature of a curve is zero then it is a ......

(d) If the binormal of a curve is constant then it is a ......

(e) If the torsion and curvature of a curve are in a constant ratio then it is a ......

(f) If the curvature of a curve is identically zero then its torsion is ......

(g) Let $\alpha = \alpha(t)$ be a curve and $\left|\frac{d\alpha}{dt}\right| = 0$ then it is .....

(h) The parametrization of a curve is ....and ....

(i) A surface $x = x(u,v)$ is called regular if $\left|\frac{\partial x}{\partial u} \times \frac{\partial x}{\partial v}\right|$ is .....

Q.4 . Calculate the Christoffel's Symbols:$\Gamma^1_{12}, \Gamma^2_{12}, \Gamma^1_{22}$ for $(ds)^2 = (dr)^2 + r^2(d\theta)^2$.

Q.5 . Find the subspace of $\mathbb{R}^3$spanned by the vectors $\{(1,-2,1),(-2,0,3),(3,-2,-2)\}$.

Q.6 . Show that every convergent sequence in a metric space is bounded.

Q.7 . State Kepler's laws of orbital motion.Prove at least one law for the closed orbits using Newton's equation of motion.

Q.8 . Find all the eigenvalues and eigenvectors of

$$4(e^{-x}y')' + (1+\lambda)e^{-x}y = 0 \qquad y(0) = 0, y(1) = 0$$

Q.9 . Solve

$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$; $\quad x > 0, t > 0 \quad u(0,t) = A, t \ge 0;\ u(x,0) = 0, x > 0,\ u$ and $\frac{\partial u}{\partial x} \to 0$ when $x \to \infty$.

Q.1 0. Use Method iteration to find a real root of the equation $x^3 + x^2 - 1 = 0$ on the interval $[0,1]$ by taking $x_0 = 0.75$.

Q.1 1. Find residue of $f(z) = \frac{1}{z(e^z-1)}$ at $z = 0$.

Q.1 2. Let $X = \{1,2,\ldots,10\}$ and $\Im = X \cup \{A \subseteq X : 1 \notin A\}$.Show that $\Im$ is a topology on X.Is $(X,\Im)$ separable,T_1, T_2, compact and connected?

Q.1 3. Let A,B,C be subgroups of a group G such that $A \subseteq B \cup C$.Show that either $A \subseteq B$ or $A \subseteq C$.

Section II

Q.1 4. Find the solution of

$$u(y) = y + \int_0^y \sin(y-t)u(t)dt.$$

Q.1 5. If $u = -\frac{cy}{r^2}, v = \frac{cx}{r^2}, w = 0$.Find out the equation of streamline.

Q.1 6. Consider electromagnetic phenomena in free space.Write Maxwell equation and prove that electric field $\bar{E}$ satisfies $\nabla^2 \bar{E} = \mu \in \frac{\partial^2 \bar{E}}{\partial t^2}$.

Q.1 7. For most earth rocks $\lambda = \mu$.Discuss values of Young's modulus and Poisson's ratio.Write down the stress-strain and the Navier equations in this case.

Q.1 8. Use Picard's method to obtain $y(0.25), y(0.5)$ and $y(1)$ where $\frac{dy}{dx} = \frac{x^2}{1+y^2}, y(0) = 0$.Perform two iterations only.

Q.1 9. Derive the Lorentz transformations for uniform,rectilinear motion of an observer S with respect to another observer S'.

Q.2 0. Find the curve connecting two given points A and B which a particle transverse while going from A to B in shortest possible time,assuming constant gravity. (Brachistochrone problem)

Section III

Q.2 1. Show that every Hilbert space is strictly convex.The space $C[a,b]$ is not strictly convex.

Q.2 2. Calculate the Ramsey number $R(3,4)$.

Q.2 3. Show that a commutative ring with identity R is a field if and only if every ideal of R is prime.

Q.2 4. (i) If $F = \mathbb{Z}_2$and

$C = \{0000000, 0101010, 1110000, 1011010, 0100101, 0001111, 1111111\}$

Find linear code and minimum distance for C.

(ii) Define orthogonal code and give an example.

Q.2 5. Show that a continuous function defined on a measurable set is measurable.

Q.2 6. Show that sum of two submodules of an R-module M is the submodule of M

generated by their union.

Q.2 7. Let Ω be the space of all functions $f: R^3 \to R$ all of whose partial derivatives (of any order) exist everywhere.Let G be the Euclidean group of R^3,that is,the set of all transfomations $T_{u,b}: a \to au + b, a \in R^3$,where $u \in O(3)$ and $b \in R^3$.For $g \in G$ and $f \in \Omega$ define $f^g$by $f^g(x,y,z) := f(x,y,z)g^{-1}$.Show that the prescription $(f,g) \mapsto f^g$gives an action of G on Ω.

Good Luck

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