M.Phil Admission Test Fall 2008

QUAID-E-AZAM UNIVERSITY, ISLAMABAD.

Mathematics , Dated: 19-8-2008, http://www.mathcity.org

Time: 9:30 a.m. to 11:00 a.m.

Dated: 19-08-2008

Note: Section I is compulsory, Section II is for Applied Mathematics candidates and Section III is for Pure Mathematics candidates.

Section I

Q.1 (a) If $0 < a < b$, determine $\lim_{n \to \infty}\left(\dfrac{a^{n+1} + b^{n+1}}{a^n + b^n}\right)$

(b) If $a > 0$, $b > 0$, show that

$$\lim_{n \to \infty}\left(\sqrt{(n+a)(n+b)} - n\right) = \frac{a+b}{2}$$

Q.2 State the Frenet-Serret apparatus for a unit speed curve. Let $\underline{\beta}(s)$ be a unit speed curve in $\mathbb{R}^3 \ni K \neq 0$ and $\tau = 0$, then $\underline{\beta}(s)$ is part of a circle of radius $1/k$.

Q.3 Find moment of inertia of a uniform rod of mass 'm' and length 'a' about a line AB perpendicular to the rod and distant $\frac{a}{5}$ from one end of the rod. Then find the moment of inertia about a line perpendicular to the rod and distant $\frac{a}{3}$ from the line AB.

Q.4 Define the convergence of an infinite series and write a necessary condition for convergence of an infinite series. Further, examine the convergence of the series $1 + \frac{1}{4^{2/3}} + \frac{1}{9^{2/3}} + \frac{1}{16^{2/3}} + \cdots$

Q.5 A tensor T_{ab} in cartesian coordinates is given by

$$T_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Transform this tensor into polar coordinates (r, θ).

P.T.O.

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Q.6 Using following tabular data approximate the value of $f(2.5)$

x	-2	0	2	3
$f(x)$	0	2	10	29

Q.7 Give an example of two non-isomorphic binary operations on $\{1,2,3,4\}$.

Q.8 (a) Find a linear mapping $f: R^2 \to R$ such that $\|f\| = 7$

(b) Show that $B(X,Y)$ bounded linear operators from a norm space X to a norm space Y cannot be finite

Q.9 Solve the following differential equation

$$(x-1)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = (x-1)^2$$

Q.10 Solve

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}, \quad -\infty < x < \infty, \quad t > 0$$

$$u(x,0) = \begin{cases} 400, & -4 \le x \le 4 \\ 0, & 4 < |x| < +\infty \end{cases}$$

Q.11 (a) Show that $u(r,\theta) = \left(r + \frac{1}{r}\right)\cos\theta$ is harmonic function.

(b) Find the principal value of $\sqrt{1+C}$

Q.12 (a) Let X be a non-empty set and $x_0 \in X$. Let $\mathcal{T} = \{X\} \cup \{A \subseteq X : x_0 \notin A\}$. Show that $\mathcal{T}$ is a topology on X.

(b) Show that every neighbourhood in a co-finite topological space is an open set.

Q.13 (a) Let $W_1 = \{(a,b,0,0) : a,b \in R\}$, $W_2 = \{(0,0,c,0) : c \in R\}$, $W_3 = \{(0,0,0,d) : d \in R\}$. Prove that $R^4 = W_1 \oplus W_2 \oplus W_3$, where R^4 is a vector space over the field R.

(b) Let $T: R^2 \to R^2$ be defined by

$$T(a,b) = (a+2b, -2a+b) \text{ and } B = \{(1,0),(0,1)\}$$

Then illustrate Cayley-Hamilton Theorem.

Section II

Q.14 Solve the following initial value problem

$$\frac{d^2u}{dx^2} = e^{2x} - \int_0^x e^{2(x-t)} \frac{du}{dt}\, dt$$

$u(0) = 0$, $u'(0) = 0$.

Q.15 The velocity components in a two-dimensional flow field are given by $V_r = \frac{\cos\theta}{r^2}$, $V_\theta = \frac{\sin\theta}{r^2}$
Find the equation of the streamline passing through the point $r = 2$, $\theta = \pi/2$

Q.16 Hookes law for homogeneous and isotropic elastic bodies is given by $\sigma_{ij} = \lambda\, \delta_{ij}\, \epsilon_{kk} + 2\mu\, \epsilon_{ij}$
Derive this law ~~form~~ from generalized Hooke's law

$$\sigma_{ij} = C_{ijkl}\, \epsilon_{kl}$$

Q.17 A rod of rest length 3 metres is moving with 90 percent of the speed of light (such that the length is in the direction of motion). Calculate the contraction in its length.

Q.18 Let the Hamiltonian operator for a system be defined as $\hat{H} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix}$ write the operator as a combination of the outer product of the base states $\{ \langle u_i |, i = 1, 2, 3 \}$. What are the allowed energy eigenvalues for this system.

Q.19 Find Hamilton's Equations of a particle which executes simple Harmonic motion

Q.20 Solve $\frac{dy}{dx} = yz + x$, $\frac{dz}{dx} = xz + y$ given that $y(0) = 1$, $z(0) = -1$. By Runge-Kutta Method of fourth order.

Section III

Q.21 Let $T: X \to X$ be a bounded linear operator on a complex inner product space X. Show that if $\langle Tx, x \rangle = 0$ for all $x \in X$ then $T = 0$.

Q.22 Show that the mean of the cauchy distribution does not exist.

Q.23 Let R be a commutative ring with identity and P an ideal in R. Prove that R/P is an integral domain if and only if P is prime ideal in R. Why a maximal ideal in R is a prime ideal?

Q.24 Indicate True or false

✓(i) Every additive abelian group is module over $\mathbb{Z}$.

×(ii) $\mathbb{Z}$ is not a module over $\mathbb{Q}$.

✓(iii) $\mathbb{Q}/\mathbb{Z}$ is module over $\mathbb{Z}$.

(iv) $\mathbb{Q}$ is not a module over $\mathbb{Z}$.

(v) $\mathbb{R}[x]$ is finitely generated module over $\mathbb{R}$.

Q.25 Show that, in their natural presentations on the appropriate projective lines, $PSL(2,5) = A_5$.

Q.26 Show that an abelian group under addition is divisible iff it is injective as $\mathbb{Z}$-module.

Q.27 Show that every set of measure zero is measurable.

$\mathbb{Q} \times \mathbb{Z} \not\to \mathbb{Z}$ Good Luck

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