

QUAID-I-AZAM UNIVERSITY, ISLAMABAD**TEST FOR M.Phil. ADMISSION FALL 2006**

Time: 9:30 a.m. to 11:00 a.m.

Dated: 22-08-06

NOTE: Attempt as many questions as you can.

- Q.1 If G is a group acting on the set Ω , and if $\alpha \in \Omega$, we define the stabilizer of α by $G_\alpha = \text{Stab}(\alpha) = \{g \in G: \alpha^g = \alpha\}$. If $\alpha, \beta \in \Omega$ and $\beta = \alpha^x$ then prove that
- $G_\beta = x^{-1} G_\alpha x$, and
 - if $\theta: \Omega \rightarrow \Omega'$ is a G -isomorphism, $\alpha^\theta = \beta$. Then $G_\beta = G_\alpha$.
- Q.2 (i) Show that $u(r, \theta) = (r + \frac{1}{r}) \cos \theta$ is harmonic.
(ii) Sketch the set of points satisfying $\text{Re}(z^2) > 4$ where $z = x + iy$
- Q.3 Let $\langle E_i \rangle$ be a sequence of measurable sets. Then prove that
- $m(\cup E_i) \leq \sum m E_i$
 - $m(\cup E_i) = \sum m E_i$ if E_n are pairwise disjoint.
- Q.4 Write down basic laws of electromagnetism. Give physical interpretation.
- Q.5 Define the term Moment of Inertia. Find the moment of inertia of a uniform mass m and length $2a$ about the perpendicular bisector.

Q.6 The kinetic energy and the potential energy of a mechanical system is given by $\frac{1}{2} m(\dot{r}^2 + \dot{\theta}^2 r^2)$ and $mgr \sin \theta$, respectively. Set up the Lagrange equations for the motion of the system.

Q.7 Test the following series for convergence

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n^2+n}}{4n+5}$

(b) $\sum_{n=4}^{\infty} \frac{\cos n}{e^{2n} + 3n - 1}$

Q.8 Determine the mean and variance of a Poisson random variable using the moment generating function method.

Q.9 Solve $\frac{d^2 u}{dx^2} = e^{2x} - \int_0^x e^{2(x-t)} \frac{du}{dt} dt$

$u(0) = 0, \quad u'(0) = 0$

Q.10 Find the solution of

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k} \frac{\partial u}{\partial t}, \quad 0 < x < 1, \quad t > 0,$$

$$u(0, t) = 1, \quad u(1, t) = 1, \quad t > 0$$

$$u(x, 0) = 1 + \sin \pi x, \quad 0 < x < 1.$$

Q.11 Approximate the smallest eigenvalue of the matrix

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Perform three iterations only.

Q.12 Find the curvature on the curve

$$x = \left(t, \frac{1}{2} t^2, \frac{1}{3} t^3 \right)$$

at the point $t = 1$

- Q.13 Explain $GL(n, \mathbb{R})$, $SL(n, \mathbb{R})$ and $M(n, \mathbb{R})$.
Moreover relate above structures with each other.
- Q.14 Let R be a commutative ring with identity and P be an ideal of R . Prove that R/P is an integral domain iff P is a prime ideal.
- Q.15 Find the polynomial which the values $(0,0)$, $(1,5)$, $(3,-3)$ and $(4,-4)$ and hence compute the value of polynomial at $x=2$.
- Q.16 Find the general solution of

$$x y'' + y' - 4y = 0$$
 of $y_1 = c_0 \sum_{n=0}^{\infty} \frac{4^n}{(n!)^2} x^n \quad |x| < \infty$
 is first solution.
- Q.17 (i) Show that in the usual \mathbb{R}^2 space the open spheres are just open discs.
 (ii) Show that in the co-finite topological space every neighbourhood is an open set.
- Q.18 (i) Show that a linear operator T is continuous if it is bounded.
 (ii) Let H be an inner product space and $x, y, z \in H$ are mutually orthogonal. Then show that

$$\|x+y+z\|^2 = \|x\|^2 + \|y\|^2 + \|z\|^2$$

Q.19 Let $f(x) = \begin{cases} x^2, & \text{if } x \leq 1 \\ x, & \text{if } x > 1 \end{cases}$

Does the Lagrange's Mean value Theorem holds for f on $[\frac{1}{2}, 2]$?

Q.20 If $W = \frac{Wa^2}{z}$. Show that the stream lines are circles; all touch the x -axis at the origin.

Q.21(a) Define Riemann curvature tensor. How many identically non-zero independent components it has in \mathbb{R}^n ?

(b) State and prove the first Bianchi identity

Q.22 What is meant by the degenerate energy eigenvalue of a system? Obtain the degenerate energy eigenvalue for a system with the Hamiltonian $H = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

in an ~~ortho~~ orthonormal basis of the state space, hence calculate ΔH for a normalized state vector. (ΔH denotes the root mean square deviation of the observable H).

Q.23 Show that every free module is projective

Q.24 Let $T: \mathbb{C}^2 \rightarrow \mathbb{C}^2$ be the linear operator defined by $T(x_1, x_2) = (x_1, 0)$. Let $B_1 = \{(1, 0), (0, 1)\}$ and $B_2 = \{(1, i), (-i, 2)\}$ be the basis of \mathbb{C}^2 . Write the matrix of T relative to the pair B_2, B_1 .