QUAID-I-AZAM UNIVERSITY
DEPARTMENT OF MATHEMATICS

Test for M.Phil. admission Fall-2005

Time: 9:30 a.m. to 11:00 a.m. **Dated: 16.08.2005**

NOTE:- **Attempt as many question as you can.**

Q.1 Show that every finite group of prime order is abelian.

Q.2 Let V be a vector space over a field F. Show that the quotient of $AGL(n, F)$ with $T(V) = \{T_{I,b} : b \in F^n\}$ is isomorphic to $GL(n, F)$.

.3 Derive the equation of continuity in Eulerian Formulation

).4 How do you get electromagnetic waves from Maxwell's equations?

.5 Define moment of a vector. Deduce from it definition of angular momentum. A particle of unit mass lying at any time t at $(t, 2t-1, t)$ moves with velocity $[2, 3, 4]$. Find the angular momentum of the particle at $t = 1$.

).6 Find Hamilton's equations of motion for a particle executing simple Harmonic motion.

P.T.O

Q.7 Classify the following integral equation

$$u(x) = 1 + \int_0^x (t-x)\, u(t)\, dt$$

verify whether $u(x) = \cos x$ is its solution or not.

Q.8. Use residues to express $f(z) = \dfrac{3z+2}{z(z-1)(z-2)}$ in partial fraction.

Q.9 Solve the following first order P.D.E

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y) z.$$

Q.10 Prove or disprove

(a) If $P(A|B) \geq P(A)$ then $P(B|A) \geq P(B)$

(b) If $P(B|A^c) = P(B|A)$ then A and B are independent.

Q.11 Show that the space $C[a,b]$ is not an inner product space and hence not a Hilbert space.

Q.12 Discuss the convergence and uniform convergence of the series $\sum f_n$ where $f_n(x)$ is given by $(nx)^{-2}$ $x \neq 0$.

Q.13 Find the matrix g_{ab} and g^{ab} for

$$x = (r\cos\theta,\ r\sin\theta) \qquad 0 \leq \theta < 2\pi$$

Q.14 Find a polynomial which interpolates the following data

x	0	1	3	4
$f(x)$	0	3	-3	0

Hence calculate $f(2)$.

Q.15 Calculate to four decimals the eigenvalue of the matrix below nearest to the number -4 with initial approximation of the eigen vector $(0,0,1)^T$. Perform three iteration only

$$\begin{bmatrix} -3 & 1 & 0 \\ 1 & -3 & -3 \\ 0 & -3 & 4 \end{bmatrix}.$$

Q.16 Compute the Frenet frame (that is, the unit tangent, the principal normal and the binormal vectors) of the unit speed helix.

Q.17 A 100 meter long train is moving at 90 percent of the speed of light. What will be its relativistic length contraction?

Q.18 Evaluate $\int \sqrt{\frac{x-1}{x^5}}\, dx$

Q.19 Solve the initial value problem $y'' - 4y' = 0$
$y(0) = 3$, $y'(0) = 8$

Q.20 Let $v_1 = \begin{bmatrix} 3 \\ 0 \\ -6 \end{bmatrix}$, $v_2 = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix}$, and $v_3 = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$
Determine if $\{v_1, v_2, v_3\}$ is a basis for $\mathbb{R}^3$ over $\mathbb{R}$.

Q.21 Let I and J be any ideals of a ring R. Show that $I+J = \{a+b : a \in I, b \in J\}$ is the smallest ideal of R containing $I \cup J$.

Q.22 Show that every divisible abelian group is an injective Z-module.

Q.23 Let R be the set of all real number and $\mathcal{J}$ consists of ϕ, R and all subsets of R of the form (a, ∞) where $a \in R$. Show that $\mathcal{J}$ is a topology on R.

Q.24 Show that a set of measure zero is measurable.

Good Luck