M.Phil Admission Test 2001

QUAID-E-AZAM UNIVERSITY, ISLAMABAD.

Mathematics , Dated: 15-08-2001, http://www.mathcity.org

Time Allowed : $1\frac{1}{2}$ Hours Dated: 15-8-2001

Attempt all the questions.

Q.1. Dynamic behaviour of a mechanical system is described by the Lagrangian $L = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}(x^2 + y^2)$. Find the Hamiltonian function for this system.

Q2. What are ordinary and singular points? Solve the equation $(1-x^2)y'' - 6xy' - 4y = 0$ near the ordinary point $x = 0$.

Q3. Solve $U_{xy} - U_y + 50 = 0$.

Q4. Show that $\alpha(s) = \left(\frac{(1+s)^{3/2}}{3}, \frac{(1-s)^{3/2}}{3}, \frac{s}{\sqrt{2}}\right)$ is a unit speed curve. Find its curvature and torsion. Is it a helix?

Q5. Solve the geodesic equation for the surface of a sphere of radius 4 meters. Name geodesics on the surface of this sphere.

Q6. Show that the function $[\mathrm{Log}(z+4)]/z^2+i$ is analytic everywhere except at the points $\pm(1-i)/\sqrt{2}$ and the portion $x \leq -4$ of real axis.

Q7. Let C be the arc bounding the square with vertices $(-1,-1)$, $(1,-1)$, $(1,1)$ and $(-1,1)$. Show that Green's Theorem is applicable to the integral $\oint_C xy^2dx - yx^2dy$. Apply the Theorem to convert the integral into a double integral and evaluate it.

Q8. Estimate the error when Simpson's $\frac{1}{3}$ rule is used to evaluate the integral $\int_0^1 \frac{dx}{1+2x^3}$, with a step size $\frac{1}{6}$.

Q9. Let X be a non-empty set and J be a collection of all subsets of X whose complements are finite and the empty set i.e. $J = \{\phi\} \cup \{A \subseteq X : A' \text{ is finite}\}$. Show that J is a topology on X.

Q10. (i) Show that a linear operator $T : X \longrightarrow Y$ is bounded iff the set $T(S) = \{T(x) : \|x\| \leq 1\}$ is a bounded subset of Y, where X and Y are normed spaces.

(ii) Prove that $C[0,1]$ w.r.t. $\|f\| = \sup_{t \in [0,1]} |f(t)|$ is a Banach space.

(iii) Give the statement of the closed graph theorem.

Q11. Define an epimorphism from $GL(2, \mathbb{Z}_3)$ to $\mathbb{Z}_3$. Find the Kernal of this homomorphism. If K denote the kernal of this homomorphism then show that $GL(2, \mathbb{Z}_3)$ is isomorphic to $\mathbb{Z}_3$.!

Q12. (i) Show that a linear transformation $T : V \longrightarrow W$ is one-one if and only if the Null space is zero.

(ii) Define eigenvector and eigenvalue. Prove that $W = \{v \in V : T(v) = \lambda v\}$ is a subspace of the vector space V. Also name the space W.

Q13. (i) Define Contineous and differentiable functions at a point.

(ii) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by

$$f(x) = 0, \quad x \in \mathbb{Q}$$
$$f(x) = 1, \quad x \in \mathbb{Q}'$$

Discuss the continiuty of the function $f(x)$.

(iii) which one is correct, Differentiability $\Rightarrow$ Continuity or Continuity $\Rightarrow$ Differentiability. Also justify incorrect implication by an example.

Q14. Find the moment generating function of the Binomial Random Variable.

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