UNIVERSITY OF GUJRAT
A WORLD CLASS UNIVERSITY

(M.A/M.Sc Part-I)
(Mathematics)
Topology and Functional Analysis

Roll No: ______________

in words: ______________

Cutting, Overwriting, Erasing, Fluid painting and use of Lead Pencil will earn no marks. Write answer of the Question No.1 and 2 on this sheet and handover it to the supervisory staff of examination within first 35 minutes.

Time Allowed: 35 Minutes ***(OBJECTIVE PART)*** ***Max. Marks: 32*** ***Sign of Supdt.***

1- a) Tick or Encircle the correct answer: ***1x4***

i) Let T : N → M be a bijective linear operator and dim N = n then dim M is

a) n + 1 b) n

c) n – 1 d) n + 2

ii) A continuous image of a compact space is

a) Connected b) Compact

c) Neither d) Both

iii) In a normed space V, $||x - y|| = 0$ iff

a) $x > y$ b) $x < y$

c) $x = y$ d) $x \neq y$

iv) Let X = {1, 2, 3} and τ = {x, φ, {1}, {3}} then τ is

a) Topology on X b) Not topology

c) T_1- Space d) Regular Space

b) Indicate True or False: ***1x8***

i) Product of two linear operators is linear operator. **True / False**

ii) The closed ball in a metric space is not closed set. **True / False**

iii) Let T : N → M be a linear operator then the Kernel of T is closed in N. **True / False**

iv) Every normal space is an inner product space. **True / False**

v) The union of topologies on X is a topology on X. **True / False**

vi) Every first countable space is second countable space. **True / False**

vii) Every T_1 - Space is T_2 – Space **True / False**

viii) A closed continuous image of a normal space is normal. **True / False**

c) Fill in the blanks meaningfully: ***1x4***

i) $(AUB)^{\perp}$ = ____.

ii) Every closed subspace of a compact space is ______________________________.

iii) Every compact subspace of a Housdorff space is ______________________________.

iv) A topology consisting all subsets of X is called______________________________.

(Continued Overleaf)

2- Give short answers the following questions: ***2x8***

i) Define Cofinite Topology.

ii) Define Normal Space.

iii) Define Finite Intersection Property.

iv) Prove that Indiscrete Space is connected.

v) Define Completely Regular Space.

vi) Define Banach Space.

vii) Define Hilbert Space.

viii) Define Equivalent Norms on a Normed Space X.

Total Marks = 68 + 32 = 100
Pass Marks = 40%

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Time Allowed : 2:25 hrs
Max. Marks : 68

Attempt any **FOUR** Questions in all. All Questions carry equal marks.

SUBJECTIVE PART

3- a) A topological space X is T_1 – Space iff each singleton set is closed. *9*

b) Let X be a topological space and A is a connected subspace of X such that $A \subseteq B \subseteq \bar{A}$. Then B is connected. In particular $\bar{A}$ is connected. *8*

4- a) Prove that every closed subset of a compact space is compact. *9*
b) Prove that every second countable space is separable. *8*

5- a) A family B of subsets of a topological space $(x, \mathcal{T})$ is a base for $\mathcal{T}$ iff *9*
i) $X = \cup B_\alpha$, that is, every point of X is in some $B_\alpha \in B$.
ii) For $B_1, B_2 \in B$ and $x \in B_1 \cap B_2$ the there is $B_3 \in B$ such that $x \in B_3 \subseteq B_1 \cap B_2$.
b) Let (x, d) be a metric space and $A \subseteq X$ then $\bar{A}$ is the smallest closed subset of X which contains A. *8*

6- a) Prove that ℓ^∞ is Banach Space. *9*
b) Prove that any two norms on a finite dimensional linear space are equivalent. *8*

7- a) Let X and Y be normed subspaces over the field F and $T : X \to Y$ be a linear operator Then T is continuous iff T is bounded. *8*
b) Let Y be a closed subspace of a Hilbert Space H. Then $H = Y \oplus y^\perp$ *9*

8- a) Let V be an complex inner space V and x, y ∈ V then

$$\langle x, y\rangle = \frac{1}{4}\left[\|x + y\|^2 - \|x - y\|^2 + i\,\|x + i\,y\|^2 - i\,\|x - i\,y\|^2\right]$$ *8*

b) Prove that the dual space of ℓ^1 is ℓ^∞ *9*