UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(Mathematics)

(M.A/M.Sc Part-I)

Mechanics

Roll No: _____

in words: _____

Cutting, Overwriting, Erasing, Fluid painting and use of Lead Pencil will earn no marks. Write answer of the Question No.1 and 2 on this sheet and handover it to the supervisory staff of examination within first 35 minutes.

Time Allowed: 35 Minutes ***(OBJECTIVE PART)*** ***Max. Marks: 32***

Sign of Supdt.

1- a) Tick or Encircle the correct answer: *1x4*

i) div r =

a) 0 b) 1

c) 2 d) None of these

ii) grad (ℓn r)

a) $\frac{1}{r}$ b) $\frac{1}{r^2}$

c) $\frac{\underline{r}}{r}$ d) $\frac{\underline{r}}{r^2}$

iii) $a_{ij}\, x'_j$ =

a) x_i b) x_j

c) x'_i d) x'_j

iv) The density of a hemisphere is

a) $\frac{m}{\frac{4}{3}\prod a^2}$ b) $\frac{m}{\frac{2}{3}\prod a^2}$

c) $\frac{m}{\prod a^3}$ d) $\frac{m}{\frac{2}{3}\prod a^3}$

b) Write true or false: *1x8*

i) $Ixy = \sum_i m_i (x_i^2 + y_j^2)$ **True / False**

ii) Theorem of Parallel axes is used to find moment of inertias of Plane bodies. **True / False**

iii) $\nabla \times \underline{r} = 3$ **True / False**

iv) $\nabla (f(r)) = f'(r)\, \hat{r}$ **True / False**

v) $\underline{W} = \underline{V} \times \underline{r}$ **True / False**

vi) A vector field $\underline{F}$ is solenoidal if $\nabla \times \underline{F} = 0$ **True / False**

vii) $\in_{iij} = 3$ **True / False**

viii) T_{ijjkk} is a tensor of rank 3. **True / False**

c) Fill in the blanks meaningfully: *1x4*

i) $(\underline{a}.\nabla)\, \underline{r}$ = _____.

ii) $\nabla . (\underline{a} \times \underline{r})$ = _____.

iii) $\in_{ijk} \delta_{jk}$ = _____.

iv) M. I of a circular disc abut a diameter whose radius is a, is _____.

(Continued Overleaf)

2- Give short answers the following questions: *2x8*

i) Write an expression for directional derivative of a scaler point function ϕ in the direction of the vector $\underline{a}$.

ii) Define an irrotational vector field and give one of its example.

iii) Evaluate, grad ($\underline{r}$. $\underline{r}$)

iv) Explain the meaning / significance of scale factors.

v) Prove that $\delta_{ij} A_i = \delta_{ji} A_i = A_j$

vi) Show that $\in_{ijk} \frac{\partial A_k}{\partial x_j} = (\text{Curl } \underline{A})_1$

vii) Find the number of degrees of freedom of a rigid body moving in a plane with one point fixed.

viii) Find moment of inertia of a uniform rigid rod of length ℓ, about an axis through one of its ends and perpendicular to the rod.

Total Marks: 68+32=100
Pass Marks = 40%

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Mechanics

Roll No: _____

Time Allowed : 2:25 hrs
Max. Marks : 68

Attempt **FOUR** Questions in all. Selecting at least one question from each section. All Questions carry equal marks.

SUBJECTIVE PART

SECTION-A

3- a) Evaluate $\nabla . [r \nabla\left(\frac{1}{r^3}\right)]$ *8*

b) Find the directional derivative of $\phi = 4e^{2x - y + z}$ at the point (1, 1, -1) in a direction towards the point (-3, 5, 6). *9*

4- a) Show that spherical polar coordinate system is an orthogonal Curvilinear Coordinate System.

Also find an expression for element of arc length in this system. *8*

b) Evaluate the integral $\int_S \underline{A} . d\underline{S}$ where $\underline{A} = zy\,\hat{i} + zx\,\hat{j} + xy\,\hat{k}$ and S is the portion of the sphere $x^2 + y^2 + z^2 = 1$ lying in the 1st octant. *9*

SECTION-B

5- a) Define Symmetric and Skew Symmetric Tensors. Prove that every 2nd rank tensor can be written as a sum of symmetric and anti symmetric tensors. *8*

b) Define Inner Product of any two tensors, prove that inner product of the tensors A_{ij} and B_{mn} is also a tensor. Write its rank. *9*

6- a) Prove Tensorially $\underline{A} \times (\underline{B} \times \underline{C}) = \underline{A} . \underline{C}\,\underline{B} - \underline{A} . \underline{B}\,\underline{C}$ *8*

b) i) Show that $\in_{ijk} \in_{ijk} = 6$. *4*

ii) Using Cartesian Formalism. Prove Curl (grad ϕ) = 0 *5*

SECTION-C

7- a) The points (a, 2a, -a), (-a, -a, a) and (a, a, a) of a rigid body have instantaneous velocities. *9*

$(\frac{\sqrt{3}}{2} v, 0, \frac{\sqrt{3}}{2} v)$, $(\frac{-v}{\sqrt{3}}, 0, \frac{-v}{\sqrt{3}})$, $(0, \frac{-v}{\sqrt{3}}, \frac{v}{\sqrt{3}})$ respectively w.r.t a rectangular coordinate system. Show that the body has line through the origion having d. es $(1, -1, -1)/\sqrt{3}$

b) Find moment of inertia of a uniform solid shpare of mass M and radious a, about axis of its symmetry and hence use it to find its M.I about a tangent line. *8*

8- a) Derive the relation [L] = [I][W] where L, I, W have their usual meanings. *8*

b) A uniform rigid rod AB moves so that A and B have velocities $\underline{u}A$ and $\underline{u}B$ at any instant.

Show that the K.E is then $T = \frac{1}{6} M (\underline{u}_A^2 + u_A . u_B + \underline{u}_B^2)$ M being the mass. *9*