(Mathematics)

(M.A/M.Sc Part-I)

Mechanics

Roll No: ______

Cutting, Overwriting, Erasing, Fluid painting and use of Lead Pencil will earn no marks. Write answer of the Question No.1 and 2 on this sheet and handover it to the supervisory staff of examination within first 35 minutes.

Time Allowed: 35 Minutes

(OBJECTIVE PART)

Max. Marks: 32

1x4

Sign of Supdt.

1- a) Tick or Encircle the correct answer:

- div r =
 - a) 0

b) 1

c) 2

d) None of these

ii) grad (ln r)

a) $\frac{1}{r}$

b) $\frac{1}{r^2}$

c) $\frac{\underline{r}}{r}$

d) $\frac{\underline{r}}{r^2}$

iii) $a_{ij} x'_{j} =$

a) x

b) x_j

c) x'_i

d) x'_i

iv) The density of a hemisphere is

a) $\frac{m}{\frac{4}{3} \prod a^2}$

b) $\frac{m}{\frac{2}{3} \prod a^2}$

c) $\frac{m}{\prod a^3}$

d) $\frac{m}{\frac{2}{3} \prod a^3}$

b) Write true or false:

1x8

i) Ixy = $\sum_{i} m_{i} (x_{i}^{2} + y_{j}^{2})$

 ∇ (f(r)) = f'(r) \hat{r}

- True / False
- ii) Theorem of Parallel axes is used to find moment of inertias of Plane bodies.
- True / False
 True / False

iii) $\nabla x \underline{r} = 3$

True / False

 $v) \qquad \underline{W} = \underline{V} \times \underline{r}$

True / False

vi) A vector field \underline{F} is solenoidal if $\nabla x \underline{F} = 0$

True / False

vii) $\epsilon_{iii} = 3$

iv)

True / False

viii) Tijikk is a tensor of rank 3.

True / False

c) Fill in the blanks meaningfully:

1x4

- i) $(\underline{\mathbf{a}}.\nabla)\underline{\mathbf{r}} = \underline{\hspace{1cm}}$
- ii) $\nabla . (\underline{\mathbf{a}} \times \underline{\mathbf{r}}) = \underline{\hspace{1cm}}$
- iii) $\in_{ijk} \delta_{jk} =$ _______
- iv) M. I of a circular disc abut a diameter whose radius is a, is ____

Write an expression for directional derivative of a scaler point function ϕ in the direction of the vector \underline{a} .

ii) Define an irrotational vector field and give one of its example.

iii) Evaluate, grad (<u>r</u>. <u>r</u>)

iv) Explain the meaning / significance of scale factors.

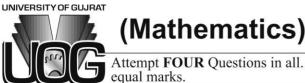
v) Prove that $\delta_{ij} A_i = \delta_{ji} A_i = A_j$

 $vi) \quad \text{Show that } \in_{ijk} \frac{\partial A_k}{\partial xj} = \left(\text{Curl } \underline{A} \right)_1$

vii) Find the number of degrees of freedom of a rigid body moving in a plane with one point fixed.

viii) Find moment of inertia of a uniform rigid rod of length ℓ , about an axis through one of its ends and perpendicular to the rod.

Total Marks: 68+32=100 Pass Marks = 40%



(M.A/M.Sc Part-I)

Roll No: _ Time Allowed:

Mechanics

8

9

8

8

Attempt FOUR Questions in all. Selecting at least one question from each section. All Questions carry

3- a) Evaluate	$\nabla \cdot [r \nabla \left(\frac{1}{3}\right)]$		8
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- b) Find the directional derivative of $\phi = 4e^{2x-y+z}$ at the point (1, 1, -1) in a direction towards the point
- **4-** a) Show that spherical polar coordinate system is an orthogonal Curvilinear Coordinate System. Also find an expression for element of arc length in this system.
 - $\int_{S} \underline{A}$ d \underline{S} where $\underline{A} = zy \hat{i} + zx \hat{j} + xy \hat{k}$ and S is the portion of the sphere $x^2 + y^2 + z^2 = 1$ lying in the 1st octant.

SECTION-B

- 5- a) Define Symmetric and Skew Symmetric Tensors. Prove that every 2nd rank tensor can be written as a sum of symmetric and anti symmetric tensors.
 - b) Define Inner Product of any two tensors, prove that inner product of the tensors A_{ii} and B_{mn} is also a tensor. Write its rank.
- **6-** a) Prove Tensorially $A \times (B \times C) = A \cdot C B - A \cdot B C$
 - b) i) Show that $\in_{iik} \in_{iik} = 6$.
 - ii) Using Cartesian Formalism. Prove Curl (grad ϕ) = 0

SECTION-C

7- a) The points (a, 2a, -a), (-a, -a, a) and (a, a, a) of a rigid body have instantaneous velocities.

$$(\frac{\sqrt{3}}{2}v, 0, \frac{\sqrt{3}}{2}v), \qquad (\frac{-v}{\sqrt{3}}, 0, \frac{-v}{\sqrt{3}}), \qquad (0, \frac{-v}{\sqrt{3}}, \frac{v}{\sqrt{3}})$$
 respectively w.r.t a rectangular coordinate

system. Show that the body has line through the origion having d. es $(1, -1, -1) / \sqrt{3}$

- b) Find moment of inertia of a uniform solid shpare of mass M and radious a, about axis of its symmetry and hence use it to find its M.I about a tangent line.
- **8-** a) Derive the relation [L] = [I][W]where L, I, W have their usual meanings.
 - b) A uniform rigid rod AB moves so that A and B have velocities <u>u</u>A and <u>u</u>B at any instant.

Show that the K.E is then
$$T = \frac{1}{6} M \left(\underline{u}_{A}^{2} + u_{A} \cdot u_{B} + \underline{u}_{B}^{2} \right) M \text{ being the mass.}$$
M.A/M.Sc-I(11/A) (MTH-IV) (N)