UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(M.A/M.Sc Part-I) (Mathematics)

Complex Analysis & Differential Geometry

Roll No: ____________

in words: ____________

Cutting, Overwriting, Erasing, Fluid painting and use of Lead Pencil will earn no marks. Write answer of the Question No.1 and 2 on this sheet and handover it to the supervisory staff of examination within first 35 minutes.

Time Allowed: 35 Minutes ***(OBJECTIVE PART)*** ***Max. Marks: 32*** ***Sign of Supdt.***

1- a) Tick or Encircle the correct answer: ***1x4***

i) The rate of turning of the binormal is denoted by

a) σ b) $\tilde{\tau}$ c) κ d) ℓ

ii) A curve traced by complex valued function $z = f(t)$ such that $f(t_1) \neq f(t_2)$ for $t_1 \neq t_2$ is called

a) Smooth Curve b) Closed Curve

c) Simple Curve d) Jordan Curve

iii) $f(z) = \frac{2}{z^2} - \frac{4}{z^7} + \frac{5}{z^5}$ has pole of order

a) Zero b) 3 c) 5 d) None of these

iv) If a function $f(z)$ involves $\overline{Z}$, then without verifying C. R equations we can say the function is

a) Harmonic b) Not analytic

c) Analytic d) None of these

b) Indicate True or False: ***1x8***

i) $\text{Cos } i z = i \text{ Cos hz}$ ………………………………………………………………… **True / False**

ii) Curvature of spherical indicatrix of tangent = $\frac{\kappa}{\sqrt{\kappa^2 + \tau^2}}$ ……………………………… **True / False**

iii) $\left| \int_C f(z)\, dz \right| \leq \left| f(z) \right| \left| dz \right|$ ………………………………………………………… **True / False**

iv) $f(z) = |z|^2$ is differentiable at $z = z_0$ ………………………………………………………… **True / False**

v) $\text{Arg}(z_1 z_2) = \text{Agr}(z_1) + \text{Arg}(z_2)$ ………………………………………………………… **True / False**

vi) $\int_C \frac{\text{Cosh } z + z^2}{(Z + 5)(Z^2 - 6)} dz = 0 \quad C: |Z| = 1$ ………………………………………… **True / False**

vii) $f(z) = e^z$ is an entire function. ……………………………………………………………… **True / False**

viii) If two function u and v are to be Harmonic Conjugate of each other then both must be constant function. ………………………………………………………………………………… **True / False**

c) Fill in the blanks meaningfully: ***1x4***

i) A value of arg z satisfying $0 \leq \arg z < 2\pi$ is called ________________________________.

ii) The necessary condition for convergence of product $\prod_{n=1}^{\alpha} (1 + a_n)$ is that ____________________.

iii) A singularity of first order is called ____.

iv) Spherical indicatrix lies on the surface of a ____.

(Continued Overleaf)

2- Answer the following questions: ***2x8***

i) Write Laplae equations in polar form.

ii) Prove that $\underline{n}' = \tau\, \underline{b} - K\underline{t}.$

iii) Prove that Log (-1 + *i*) = $\frac{1}{2}\ \ell n\ 2 + \frac{3\pi}{4}\ i.$

iv) State Mittag Leffer's Expansion Theorem.

v) Write symbols of the fundamental magnitudes of the first and second order

vi) Find radius of Convergence $\sum_{n=0}^{\alpha} \frac{z^n}{n!}$

vii) State Laurent's Series and name its parts.

viii) Writ T^{-1} (W) of the transformation $T(Z) = \frac{Z}{Z+4}$

Total Marks = 68 + 32=100
Pass Marks: 40%

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(Mathematics)

Complex Analysis & Differential Geometry

Roll No: ____________

Time Allowed : 2:25 hrs
Max. Marks : 68

Attempt **FOUR** Questions in all. Select **TWO** Questions from **Section A** and **TWO** Questions from **Section B.** All Questions carry equal marks.

SUBJECTIVE PART

SECTION-A

3- a) Find the fixed points and normal form of bilinear transformation $w = \frac{3\,iz + 1}{z + i}$ *8*

b) Show that the function $f(z) = \text{Exp}\, C\left(z + \frac{1}{z}\right)$ can be expanded as Laurent's Series. $\sum_{-\alpha}^{\alpha} a_n Z^n$ for $|z| > 0$

Where $a_n = \frac{1}{2\pi}\int_0^{2\pi} e^{2C\cos\theta} \cos n\theta \, d\theta$ *9*

4- a) Prove by Contour Integration $\int_{-\alpha}^{\alpha} \frac{a\cos x + x\sin x}{x^2 + a^2}\, dx = 2\pi e^{-a}, a > 0$ *9*

b) Prove that $\int_0^{2\pi} \frac{d\theta}{(a + b\cos\theta)^2} = \frac{2\pi a}{(a^2 - b^2)^{3/2}}$ *9*

5- a) Prove that every bilinear transformation with two fixed points α, β can be put in the normal form

$$\frac{W - \alpha}{W - \beta} = K\frac{(Z - \alpha)}{(Z - \beta)}$$ *9*

b) State and prove Cauchy's Integra Formula. *8*

SECTION-B

6- a) Prove that the position vector f the current point on a curve satisfies the differential equation,

$$\frac{d}{ds}\left\{\sigma\frac{d}{ds}\left(\ell \quad \frac{d^2 r}{ds^2}\right)\right\} + \frac{d}{ds}\left(\frac{6}{\ell} \quad \frac{dr}{ds}\right) + \frac{\ell}{6}\frac{d^2 r}{ds^2} = 0$$ *9*

b) If ψ is the angle between two directions determined by; $Pdu^2 + Qdudv + Rdv^2 = 0$ then show that

$$\tan\psi = \frac{H\sqrt{Q^2 - 4PR}}{ER - FQ + GP}$$ *8*

7- a) The envelope of the plane $\ell x + my + nz = p$ where $p^2 = a^2\ell^2 + b^2m^2 + c^2n^2$ is an ellipse. *9*

b) Derive Serret Frent Formulae. *8*

8- a) Prove that $f(z) = \begin{cases} \frac{x^3(1 + i) - y^3(1 - i)}{x^2 + y^2} & \text{when } z \neq 0 \\ 0 & \text{when } z = 0 \end{cases}$ Cauchy's Riemann equations

are satisfied at origin but $f'(0)$ fails to exist. *9*

b) Find the curvature and torsion of the curve x = a Cos u, y = a Sin u, z = a Cos u *8*