UNIVERSITY OF GUJRAT
A WORLD CLASS UNIVERSITY

(Mathematics)

(M.A/M.Sc Part-I)
Algebra

Roll No: ______________
in words: ______________

Cutting, Overwriting, Erasing, Flood painting and use of Led Pencil will earn no marks. Write answer of the Question No.1 and 2 on it and handed over to the supervisory staff of examination within 35 minutes.

Time Allowed: 35 Minutes ***(OBJECTIVE PART)*** ***Max. Marks: 32*** ***Sign of Supdt.***

1- a) Tick or Encircle the correct answer: *1x4*

i) If H_1, H_2 are subgroups of a group G. Then $H_1 \cup H_2$ is a subgroup of G if and only if

a) $H_1 \cap H_2 = \phi$ b) $H_1 \cap H_2 = H_1$

c) $H_1 \cup H_2 = H_1$ d) Either (b) or (c) is true.

ii) Which of the following is not a prime ideal of the ring $\mathbb{Z}$ of integers

a) $2\mathbb{Z}$ b) $3\mathbb{Z}$ c) $15\mathbb{Z}$ d) $7\mathbb{Z}$

iii) Let W = < (1, 2, 3), (0, 1, 0)> be a subspace of IR^3 (IR). Then dim of W is

a) 1 b) 3 c) 2 d) 6

iv) The set $\mathbb{Z}_8$ = {0, 1, 2, 3, 4, 5, 6, 7} under addition and multiplication modulo of form.

a) Integral Domain b) Division Ring

c) Field d) Cumulative Ring

b) Indicate True or False: *1x8*

i) ($3\mathbb{Z}$, +, ·) is a ring without identity. **True / False**

ii) The generators of the cyclic group ($\mathbb{Z}$, +) are 1 and -1. **True / False**

iii) If r divides the order of a group G. Then G has a subgroup of order r iff r is even. **True / False**

iv) Every subgroup of an abelian group is normal in the group. **True / False**

v) The ring $\mathbb{Z}_6$ = {0, 1, 2, 3, 4, 5} with respect to addition and multiplication module 6 contain no zero divisor. **True / False**

vi) The set S = {(1, 3), (4, 12), (1, 5)} is linearly depended subset of IR^2 (R). **True / False**

vii) Every basis is maximal linearly independent set. **True / False**

viii) Let T: $IR^2 \rightarrow R$ be defined by T(x, y) = x + y - 1 then T is linear transformation. **True / False**

c) Fill in the blanks meaningfully: *1x4*

i) A group homomorphism ϕ is injective if and only if ______________________

ii) A commutative division ring is ______________________

iii) Centre of an abelian group is ______________________

iv) If R is a commutative ring such that $a \neq 0$ and $\exists\ b \in R$ s.t ab = 0, then $a \in IR$ is called

______________________.

(Continued Overleaf)

2- Give short answers the following questions: ***2x8***

i) Define Rank and Nullity of a Vectorspace.

ii) Let $G = \langle a : a^{15} = e \rangle$. Write all the subgroups of G.

iii) Differentiate between Centralizer and Normalizer of a subset of a Group.

iv) If H and K are two sylow p-subgroups of a group G, describe the relation between H and K.

v) Find the Eigen values of $\begin{pmatrix} 1 & 0 \\ 1 & 3 \end{pmatrix}$

vi) What is PRINCIPAL IDEAL and PRINCIPAL IDEAL RING? Give example of each.

vii) If $T : IR^4 \rightarrow IR^4$ is a linear Transformation and dim (R(T)) = 4. What is dimension of N(T)?

viii) Find the characteristic of the ring $\mathbb{Z}_3 \times \mathbb{Z}_2$.

Total Marks: 68 + 32 = 100
Pass Marks: 40%

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Roll No: ______________

Time Allowed : 2:25 hrs
Max. Marks : 68

Attempt **FOUR** Questions in all. Select **TWO** Questions from **Section A** and **TWO** Questions from **Section B.** All Questions carry equal marks.

SUBJECTIVE PART

SECTION-A

3- a) Let G be a group, H a subgroup and K a normal subgroup of G then show that $H/_{H\cap K} \cong HK/_K$ *9*

b) Prove that a group of prime order is abelian. Is a group of prime order cyclic group? Verify your answer. *4,1,3*

4- a) Define a Permutation and a Transposition. Write all subgroups of S_3. Specify which one is normal and which one is not and why? *2,3,3*

b) Let H and K be subgroups of a group G. Prove that the product HK is a subgroup of G if and only if HK = KH. *9*

5- a) If G is a group of order n divisible by a prime p. Prove that the number of sylow p-subgroups of G is 1 + tp, where t ∈ N U {0} and 1 + tp divides n. *9*

b) Prove that Centre of a finite p-group is non-trivial. *8*

SECTION-B

6- a) Show that ring $\mathbb{Z}$ of integers is a principle ideal ring. *9*

b) Let R, R′ be rings and $\phi : R \rightarrow R'$ be a ring hamomorphism. Then prove that $R/_{\ker\phi} \cong \text{Im}\phi$. *8*

7- a) Suppose T : V → W is a linear transformation from a finite dimensional vector space V into a vector space W. Show that dim V = Rank of T + Nullity of T. *9*

b) If V and W are of dimension m and n respectively over F. Then prove that Hom (V, W) is of dimension ‘mn’ over F. *8*

8- a) Define Similar Matrices. Prove that any two similary matrices have same eigen values. *8*

b) Check the matrix $A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$

for diagonalizability. If yes, find a matrix Q such that $Q^{-1}AQ$ is a diagonal matrix. *9*