

(Mathematics)

Cutting, Overwriting, Erasing, Flood painting and use of Led Pencil will earn no marks. Write answer of the Question No.1 and 2 on it and handed over to the supervisory staff of examination within 35 minutes.

Time Allowed: 35 Minutes

(OBJECTIVE PART)

Max. Marks: 32

*Sign of
Supdt.*

1- a) Tick or Encircle the correct answer:

1x4

- i) If H_1, H_2 are subgroups of a group G . Then $H_1 \cup H_2$ is a subgroup of G if and only if
- a) $H_1 \cap H_2 = \phi$ b) $H_1 \cap H_2 = H_1$
- c) $H_1 \cup H_2 = H_1$ d) Either (b) or (c) is true.
- ii) Which of the following is not a prime ideal of the ring \mathbb{Z} of integers
- a) $2\mathbb{Z}$ b) $3\mathbb{Z}$ c) $15\mathbb{Z}$ d) $7\mathbb{Z}$
- iii) Let $W = \langle (1, 2, 3), (0, 1, 0) \rangle$ be a subspace of \mathbb{R}^3 (\mathbb{R}). Then dim of W is
- a) 1 b) 3 c) 2 d) 6
- iv) The set $\mathbb{Z}_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ under addition and multiplication modulo of form.
- a) Integral Domain b) Division Ring
- c) Field d) Cumulative Ring

b) Indicate True or False:

1x8

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|-------|--|--------------|
| i) | $(3\mathbb{Z}, +, \cdot)$ is a ring without identity. | True / False |
| ii) | The generators of the cyclic group $(\mathbb{Z}, +)$ are 1 and -1. | True / False |
| iii) | If r divides the order of a group G . Then G has a subgroup of order r iff r is even. | True / False |
| iv) | Every subgroup of an abelian group is normal in the group. | True / False |
| v) | The ring $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ with respect to addition and multiplication module 6 contain no zero divisor. | True / False |
| vi) | The set $S = \{(1, 3), (4, 12), (1, 5)\}$ is linearly depended subset of $\mathbb{R}^2(\mathbb{R})$. | True / False |
| vii) | Every basis is maximal linearly independent set. | True / False |
| viii) | Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by $T(x, y) = x + y - 1$ then T is linear transformation. | True / False |

c) Fill in the blanks meaningfully:

 1×4

- i) A group homomorphism ϕ is injective if and only if _____
- ii) A commutative division ring is _____
- iii) Centre of an abelian group is _____
- iv) If R is a commutative ring such that $a \neq 0$ and $\exists b \in R$ s.t $ab = 0$, then $a \in R$ is called _____

(Continued Overleaf)

2- Give short answers the following questions:

2x8

- i) Define Rank and Nullity of a Vectorspace.

- ii) Let $G = \langle a : a^{15} = e \rangle$. Write all the subgroups of G .

- iii) Differentiate between Centralizer and Normalizer of a subset of a Group.

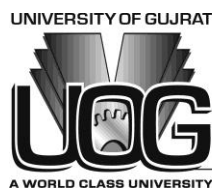
- iv) If H and K are two sylow p -subgroups of a group G , describe the relation between H and K .

- v) Find the Eigen values of $\begin{pmatrix} 1 & 0 \\ 1 & 3 \end{pmatrix}$

- vi) What is PRINCIPAL IDEAL and PRINCIPAL IDEAL RING? Give example of each.

- vii) If $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ is a linear Transformation and $\dim (R(T)) = 4$. What is dimension of $N(T)$?

- viii) Find the characteristic of the ring $\mathbb{Z}_3 \times \mathbb{Z}_2$.

**(Mathematics)****(M.A/M.Sc Part-I)****Algebra**

Roll No: _____

Time Allowed : 2:25 hrs
Max. Marks : 68Attempt **FOUR** Questions in all. Select **TWO** Questions from **Section A** and **TWO** Questions from **Section B**. All Questions carry equal marks.**SUBJECTIVE PART****SECTION-A**

- 3- a) Let G be a group, H a subgroup and K a normal subgroup of G then show that $H / H \cap K \cong HK / K$ 9
 b) Prove that a group of prime order is abelian. Is a group of prime order cyclic group? Verify your answer. 4,1,3
- 4- a) Define a Permutation and a Transposition. Write all subgroups of S_3 . Specify which one is normal and which one is not and why? 2,3,3
 b) Let H and K be subgroups of a group G . Prove that the product HK is a subgroup of G if and only if $HK = KH$. 9
- 5- a) If G is a group of order n divisible by a prime p . Prove that the number of sylow p -subgroups of G is $1 + tp$, where $t \in \mathbb{N} \cup \{0\}$ and $1 + tp$ divides n . 9
 b) Prove that Centre of a finite p -group is non-trivial. 8

SECTION-B

- 6- a) Show that ring \mathbb{Z} of integers is a principle ideal ring. 9
 b) Let R, R' be rings and $\phi : R \rightarrow R'$ be a ring homomorphism. Then prove that $R / \ker \phi \cong \text{Im} \phi$. 8
- 7- a) Suppose $T : V \rightarrow W$ is a linear transformation from a finite dimensional vector space V into a vector space W . Show that $\dim V = \text{Rank of } T + \text{Nullity of } T$. 9
 b) If V and W are of dimension m and n respectively over F . Then prove that $\text{Hom}(V, W)$ is of dimension ' mn ' over F . 8
- 8- a) Define Similar Matrices. Prove that any two similar matrices have same eigen values. 8
 b) Check the matrix $A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$ for diagonalizability. If yes, find a matrix Q such that $Q^{-1}AQ$ is a diagonal matrix. 9