UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(Mathematics)

(M.A/M.Sc Part-I)

Real Analysis

Roll No: ______________

in words: ______________

Cutting, Overwriting, Erasing, Fluid painting and use of Lead Pencil will earn no marks. Write answer of the Question No.1 and 2 on this sheet and handover it to the supervisory staff of examination within first 35 minutes.

Time Allowed: 35 Minutes ***(OBJECTIVE PART)*** ***Max. Marks: 32*** ***Sign of Supdt.***

1- a) Tick or Encircle the correct answer: ***1x4***

i) $\lim_{n \to \infty} (n)^{1/n}$

a) 0 b) $\frac{1}{2}$ c) 1 d) ∞

ii) Every subsequence of a convergent sequence is convergent and converges to the _______ limit.

a) Same b) Different c) Infinite d) None of these

iii) If f is _______________ on [a, b] then f ∈ IR (α) on [a, b]

a) Bounded b) Unbounded c) Continuous d) Monotonic

iv) $\int_2^\infty \frac{dx}{x(\ell ogx)^p}$ converges as

a) $p < 1$ b) $p = 1$ c) $p > 1$ d) $p \geq 1$

b) Indicate True or False: ***1x8***

i) The least upper bound of a set may or may not belong to the set. **True / False**

ii) In a metric space (X,d), every convergent sequence, is a Cauchy Sequence. **True / False**

iii) π is a rational number Set $A = \left\{\frac{(-1)^n}{n} \forall n = 1, 2, 3, \ldots\right\}$ Inf A = 0 ∉ A. **True / False**

iv) For a monotonic decreasing sequence $\{u_n\}$ $u_n \leq u_{n+1} \forall n = 1, 2, 3, \ldots$ **True / False**

v) The set Q of rational numbers are never unbounded. **True / False**

vi) For an infinite p series $\sum \frac{1}{n^p}$, dgs if $p \leq 1$. **True / False**

vii) The continuous image of a compact space is continuous. **True / False**

viii) F is a function of bounded variation so is $|f|$ **True / False**

c) Fill in the blanks meaningfully: ***1x4***

i) A derivable function f at a point x ∈ [a, b] is also _________________________________ at that point.

ii) The function $f(x) = [x]$ is called ____.

iii) For a metric space (X,d), the union of any collection of open sets in X is ______________________.

iv) The improper Integral $\int_0^\infty \frac{\text{Sinx } dx}{x}$ converges but ____.

(Continued Overleaf)

2- Give short answers the following questions: *2x8*

i) What is an irrational number? ______

ii) Define an Upper Bound of a Set.

iii) Define Cauchy's Real Sequence.______

iv) State the ratio test of series $\sum u_n$.______

v) Define uniform continuity of a function. ______

vi) State Lagrange's Mean Value Theorem. ______

vii) Define improper integral of First Kind.

viii) State the Fundamental Theorem of Calculus.

Total Marks: 68 + 32 = 100
Pass Marks: 40%

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(M.A/M.Sc Part-I)

(Mathematics) **Real Analysis**

Roll No: ____________

Time Allowed : 2:25 hrs
Max. Marks : 68

Attempt **FOUR** Questions in all. Select **TWO** Questions from **Section A** and **TWO** Questions from **Section B.** All Questions carry equal marks.

SUBJECTIVE PART

SECTION-A

3- a) If x, y ∈ IR and x > 0 then there exists some natural number n such that nx > y. *9*

b) To prove that every Cauchy Sequence is bounded but converse may not be true. *8*

4- a) Find the radious and the interval of convergence of the power series $\sum \frac{\lfloor n \; x^n}{\lfloor 2n}$ *9*

b) A function f is continuous at x = c ∈ I then $|f|$ is also continuous but converse may not be true. *8*

5- a) If 0 < a < b ∀ a, b ∈ IR^+ then prove that $1 - \frac{a}{b} < \ell og \frac{b}{a} < \frac{b}{a} - 1$ *9*

b) $f(x, y) = \begin{cases} \frac{xy^3}{x^2+y^6} & (x \quad y) \neq (0 \quad 0) \\ 0 & (x \quad y) = (0 \quad 0) \end{cases}$ *8*

SECTION-B

6- a) If f ∈ R [a, b] then show that $|f|$ ∈ R [a, b] and $\left| \int_a^b f \right| \leq \int_a^b |f|$. *9*

b) A function $f : [a, b] \rightarrow IR$ is Riemann Stieltjes Integrable w.r.t α iff ∀ ε > 0, ∃ a partition P of [a, b] such that U (P, f, α) – L (P, f, α) < ε. *8*

7- a) Show that the series $\sum f_n(x) = \frac{n^2x}{1+n^4x^2}$ is not uniformly convergent on [0, 1]. *9*

b) Show that series for which $f_n(x) = \frac{1}{1 + nx}$ can be integrated term by term on [0, 1] *8*

8- Examine the convergence of the following improper integrals. *9,8*

a) $\int_0^{\pi/2} \frac{\text{Sinx}}{x^P}$ b) $\int_0^{\infty} \frac{dx}{x^{1/3}(1 + x^{1/2})}$