UNIVERSITY OF GUJRAT Mathematics)

(M.A/M.Sc Part-I) Real Analysis

Roll No: _

Cutting, Overwriting, Erasing, Fluid painting and use of Lead Pencil will earn no marks. Write answer of the Question No.1 and 2 on this sheet and handover it to the supervisory

Time Allowed: 35	Minutes
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(OBJECTIVE PART)

Max. Marks: 32

Sign of Supdt.

1- a) Tick or Encircle the correct answer:

 $\lim_{n \to \infty} (n)^{1/n}$

a) 0

i)

- b) $\frac{1}{2}$
- c) 1
- d) ∞

Every subsequence of a convergent sequence is convergent and converges to the _____ ii)

- a) Same
- b) Different
- c) Infinite
- d) None of these

1x4

If f is _____ on [a, b] then $f \in IR(\alpha)$ on [a, b] iii)

- a) Bounded
- b) Unbounded
- c) Continuous
- d) Monotonic

iv) $\int_{2}^{\infty} \frac{dx}{x(\log x)^{P}}$ converges as

a) p < 1

vi)

- b) p = 1
- c) p > 1
- d) $p \ge 1$

b) Indicate True or False:

1x8

i) The least upper bound of a set may or may not belong to the set.

True / False

In a metric space (X,d), every convergent sequence, is a Cauchy Sequence. ii)

True / False

 π is a rational number Set A = $\left\{\frac{(-1)^n}{n} \forall n = 1, 2, 3, \ldots\right\}$ Inf A = 0 \notin A. iii)

True / False

iv) For a monotonic decreasing sequence $\{u_n\}$ $u_n \le u_{n+1} \forall n = 1, 2, 3, \dots$ True / False

The set Q of rational numbers are never unbounded. v)

True / False

True / False

For an infinite p series $\sum \frac{1}{n}$, dgs if $p \le 1$. The continuous image of a compact space is continuous. vii)

True / False

viii) F is a function of bounded variation so is |f| True / False

c) Fill in the blanks meaningfully:

1x4

A derivable function f at a point $x \in [a, b]$ is also ______ at that point. i)

The function f(x) = [x] is called _____ ii)

For a metric space (X,d), the union of any collection of open sets in X is iii)

The improper Integral $\int_{0}^{\infty} \frac{\sin x \, dx}{x}$ converges but ______ iv)

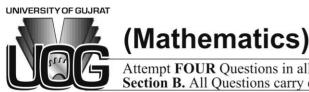
	2- Give snort answers the following questions:	2x8
	i) What is an irrational number?	
	/	
	ii) Define an Upper Bound of a Set.	
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<u>,,,, </u>		
iii)	Dafina Cauchy's Paul Saguanca	
111)	Define Cauchy's Real Sequence	
:>	State the matic test of conice \(\sigma \)	
iv)	State the ratio test of series $\sum u_n$.	
v)	Define uniform continuity of a function.	
vi)	State Lagrange's Mean Value Theorem.	
vii)	Define improper integral of First Kind.	
viii)	State the Fundamental Theorem of Calculus.	

Total Marks: 68 + 32 = 100

Pass Marks: 40%

8

9,8



(M.A/M.Sc Part-I) **Real Analysis**

Roll No:

Attempt FOUR Questions in all. Select TWO Questions from Section A and TWO Questions from Section B. All Questions carry equal marks.

SUBJECTIVE PART

SECTION-A

- 3- a) If $x, y \in IR$ and x > 0 then there exists some natural number n such that nx > y. 9 b) To prove that every Cauchy Sequence is bounded but converse may not be true. 8
- 4- a) Find the radious and the interval of convergence of the power series $\sum \frac{\lfloor \frac{n}{n} \rfloor^n}{2n}$
 - b) A function f is continuous at $x = c \in I$ then |f| is also continuous but converse may not be true. 8

5- a) If
$$0 < a < b \ \forall a, b \in IR^+$$
 then prove that $1 - \frac{a}{b} < \log \frac{b}{a} < \frac{b}{a} - 1$

b) $f(x, y) = \begin{cases} \frac{xy^3}{x^2 + 6} & (x + y) \neq (0 + 0) \\ 0 & (x + y) = (0 + 0) \end{cases}$

8

SECTION-B

- **6-** a) If $f \in \mathbb{R}$ [a, b] then show that $|f| \in \mathbb{R}$ [a, b] and $|\int_{-\infty}^{b} f| \le \int_{-\infty}^{b} |f|$. 9
 - b) A function $f: [a, b] \to IR$ is Riemann Stieltjes Integrable w.r.t α iff $\forall \epsilon > 0$, \exists a partition P of [a, b] such that $U(P, f, \alpha) - L(P, f, \alpha) \le \varepsilon$.
- 7- a) Show that the series $\sum f_n(x) = \frac{n^2 x}{1 + n^4 x^2}$ is not uniformly convergent on [0, 1].
 - b) Show that series for which $f_n(x) = \frac{1}{1 + nx}$ can be integrated term by term on [0, 1] 8
- **8-** Examine the convergence of the following improper integrals.

a)
$$\int_{0}^{\pi/2} \frac{\sin x}{x}$$
 b)
$$\int_{0}^{\infty} \frac{dx}{x^{1/3} (1 + x^{1/2})}$$

M.A/M.Sc-I(11/A) (MTH-I) (N)