University of Sargodha

M.A/M. Sc. Part-1/Composite, 2nd-A/2015

Mathematics: V   Topology & Functional Analysis

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

Objective Part (Compulsory)

Q1. Write short answers of the following: **(10x2=20)**

i. Prove that an open ball centered at a limit point x of a subset A in a metric space X contains infinitely many points of set A.

ii. Define the product topology and write its base.

iii. Prove that every metric space is a topological space.

iv Define reflexive space.

v. Show that every normed space is a metric space.

vi. State the condition for a normed space to be an inner product space and give an example.

vii. Definean orthonormal system in an inner product space.

viii. Prove that T_2- axiom implies T_1-axiom as well as T_0-axiom.

ix. Show that R^2 is a Banach space.

x. Define completion of a metric space.

Subjective Part

Q2. (a)Prove that if a normed space X has the property that the closed unit ball $M = \{x \mid \|x\| \leq 1\}$ is compact, then X is finite dimensional. 10

(b)Show that he spacel^p with $1 \leq P < \infty$is separable. 10

Q3. (a) Let X and Y be topological spaces. Show that a function f: X→ Y is continuous if and only if $f(Cl(A)) \subseteq Cl(f(A))$ 10

(b) Show that closed subspaces of Lindelof space are also Lindelof. 10

Q4. (a) Show that in a Hausdorff space, any sequence converges to at most one limit point. 10

(b) Prove that a topological space (X,τ) is normal if and only if for any closed set A and an open set U containing A, there exists at one open set V containing A such that $A \subseteq V \subseteq Cl(V) \subseteq U$. 10

Q5. (a) Let X be aT_1- space, prove that X is countably compact if and only if X satisfies Bolzano-Weierstrass property. 10

(b) Prove that R^n is a complete metric space. 10

Q6. (a) Let Y be proper closed subspace of a normed space X and a$\in (0,1)$, there is an $x_a \in X$ such that $\|x_a\| = 1$ and $\|x - x_a\| \geq a$ for all $x \in Y$. 10

(b) Prove that dual of R^n is R^n. 10