University of Sargodha

M.A/M.Sc Part- 1/Composite, 2nd-A/2014

Mathematics: V Topology & Functional Analysis

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

Objective Part

Q.1. Write short answer of the following. **(10 ×2=20)**

i. Define discrete topology.
ii. Define sub-base for a topology.
iii. Let X be a topological space and $A, B \subset X$, show that $A \subset B \Rightarrow \overline{A} \subset \overline{B}$.
iv. Differentiate between dual space and second dual space.
v. Define equivalent norms.
vi. If (x_n) converges to $x \in X$, then show that every subsequence (x_{n_k}) also converges to $x \in X$.
vii. Define Completely Regular space.
viii. Find the interior of set of rational numbers over usual topology on R (set of real numbers).
ix. Show that every Cauchy sequence is bounded.
x. Show that the operator $T : X \to X$ by $Tx(t) = x'(t)$ is linear.

Subjective Part

Q.2. (a) Show that the space $C[a, b]$ is complete. **(10)**
(b) Let X and Y be two topological spaces, $f : X \to Y$ is continuous iff, for every subset A of X, $f(\overline{A}) \subset \overline{f(A)}$. **(10)**

Q.3. (a) Let (X, d) be a metric space, $A \subset X$ is dense if and only if A has non-empty intersection with any open subset of X. **(10)**
(b) Show that in a finite dimensional normed space X, any subspace $Y \subset X$ is compact iff Y is closed and bounded. **(10)**

Q.4. (a). Show that any two norms of a finite dimensional space are equivalent. **(10)**
(b) State and prove Cauchy-Schwartz inequality for inner product space. **(10)**

Q.5. (a) Let A and B be disjoint compact subsets of a Hausdorff space X. Show that there exists disjoint open sets G and H such that $A \subset G$ and $B \subset H$. **(10)**
(b) Suppose that A and B are connected sets which are not separated. Show that their union $A \cup B$ is also connected. **(10)**

Q.6. (a) Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, X, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$ be a topology on X, let $A = \{b, c, d\}$. Find interior, exterior, closure, boundary and limit points of A. **(10)**
(b) Show that every metric space is T_2 space. **(10)**

Q.7. (a) State and prove F. Riesz's Lemma. **(10)**
(b) Show that the compact subset of a metric space is closed and bounded. **(10)**
