University of Sargodha

M.A/M.Sc Part-1 / Composite, 2nd-A/2013

Mathematics: V Topology & Functional Analysis

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

(Objective Part)

Q. 1	Answer the following short questions.	20
(i)	Define cofinite topology.	
(ii)	For any subset A of a discrete topological space X, show that derived set of A is empty.	
(iii)	Show that the plane $\Re^2$ satisfies the second axiom of countability.	
(iv)	Define metrizable topology.	
(v)	Prove that every T_3 space is also a Hausdroff space.	
(vi)	For any metric space (X, d) define neighbourhood of a point.	
(vii)	Show that a metric d induced by a norm on a normed space X satisfies $d(\alpha x, \alpha y) = \|\alpha\|d(x, y)$.	
(viii)	Define Schauder basis.	
(ix)	If normed space X is finite dimensional, then show that every linear operator on X is bounded.	
(x)	For any bounded linear operator T show that $x_n \to x$ implies $Tx_n \to Tx$.	
	(Subjective Part)	
Q. 2	(a) Show that a function $f : X \to Y$ is continouos if and only if the inverse image of every closed subset of Y is closed subset of X.	10
	(b) If β be the class of subsets of a non empty set X.Then the topology τ on X generated by β is the intersection of topologies on X which contain β.	10
Q.3	(a) Let X be a T_1 space which satisfies the first axioms of countability then show that if $p \in X$ is an accumulation point of $A \subset X$ then there exists a sequence of distinct terms in A converging to p.	10
	(b) Let A be any subset of a topological space (X, τ) and let τ_A be the relative topology on A.Then show that A is τ connected iff A is τ_A connected.	10
Q.4	(a) State and prove Cantor's Intersection theorem.	10
	(b) Show that set of real numbers is complete.	10
Q. 5	(a) Show that in a finite dimensional normed space X, any subset M of X is compact if and only if M is closed and bounded.	10
	(b)Prove that the property of being a Hausdroff space is hereditry	10
Q. 6	(a) If Y is a Banach space, then show that $B(X, Y)$ is also a Banach space.	10
	(b)State and prove minimizing vector theorem.	10
Q. 7	(a) Show that a complete metric space is of second category.	10
	(b)Show that the dual space of $\Re^n$ is $\Re^n$.	10