University of Sargodha

M.A/M.Sc Part-1 / Composite, 2nd-A/2012

Mathematics: V   Topology & Functional Analysis

Maximum Marks: 80 *Subjective Part* **Time Allowed: 2:40 Hours**

Note: **Question No.2 is compulsory. Attempt any three out of the remaining questions.**

Q2. Answer the following questions.

(i) Show that every normed space is a metric space.

(ii) Give example of a reflexive space.

(iii) State the condition for a normed space to be an inner product space and give an example.

(iv) For a subset A of a Hilbert space H show that $A^{\perp}$ is closed subspace of H.

(v) Prove that every metric space is a Haudorff space. 6

(vi) Define reflexive space.

(vii) Write all closed subsets of $X = \{a, b, c\}$ with respect to its indiscrete topology

(viii) Define completion of a metric space.

(ix) Show that any two equivalent norms on N define the same topology on N.

(x) Prove Bessel's inequality.

Q3. (a) Let X and Y be topological spaces. Show that a function $f: X \to Y$ is continuous if and only if

$f(Cl(A)) \subseteq Cl(f(A))$ 2 10

(b) Show that closed subspaces of Lindelof space are also Lindelof. 6 10

Q4. (a) Prove that any two norms defined on a finite dimensional linear space are equivalent. 10

(b) Prove that a topological space (X, τ) is normal if and only if for any closed set A and an open set U containing A, there exists at one open set V containing A such that

$A \subset V \subseteq Cl(V) \subseteq U$ 10

Q5. (a) Let X be a T_1 space, prove that X is countably compact if and only if X satisfies Belzano Weierstrass property 10

(b) Prove that R^n is a complete metric space 10

Q6. (a) Let Y be proper closed subspace of a normed space X and $a \in (0,1)$, there is an $x_a \in X$ such that $\|x_a\| = 1$ and $\|x - x_a\| \geq a$ for all $x \in Y$ 10

(b) Prove that dual of R^n is R^n 10

Q7. (a) Prove that every Hilbert space is reflexive 10

(b) State and prove minimizing vector theorem 10