

University of Sargodha

M.A/M.Sc Part-1 / Composite, 2nd-A/2012

Mathematics: V Topology & Functional Analysis

Maximum Marks: 80

Subjective Part

Time Allowed: 2:40 Hours

Note: Question No.2 is compulsory. Attempt any three out of the remaining questions.

Q2. Answer the following questions.

- (i) Show that every normed space is a metric space.
- (ii) Give example of a reflexive space.
- (iii) State the condition for a normed space to be an inner product space and give an example.
- (iv) For a subset Λ of a Hilbert space H show that Λ^\perp is closed subspace of H .
- (v) Prove that every metric space is a Hausdorff space.
- (vi) Define reflexive space.
- (vii) Write all closed subsets of $X = \{a, b, c\}$ with respect to its indiscrete topology.
- (viii) Define completion of a metric space.
- (ix) Show that any two equivalent norms on \mathbb{N} define the same topology on \mathbb{N} .
- (x) Prove Bessel's inequality.

(6)

Q3. (a) Let X and Y be topological spaces. Show that a function $f: X \rightarrow Y$ is continuous if and only if

$$f(C_l(\Lambda)) \subset C_l(f(\Lambda))$$

- (b)** Show that closed subspaces of Lindelof space are also Lindelof.

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Q4. (a) Prove that any two norms defined on a finite dimensional linear space are equivalent.

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- (b) Prove that a topological space (X, τ) is normal if and only if for any closed set A and an open set U containing A , there exist at one open set V containing A such that

$$A \subset V \subset \text{Cl}(V) \subset U$$

Q5. (a) Let X be a T_1 -space, prove that X is compact if and only if X satisfies

Bolzano-Weierstrass property



- (b) Prove that \mathbb{R}^n is a complete metric space



Q6. (a) Let Y be proper closed subspace of a normed space X and $a \in (0, 1)$, there is an $x_a \in X$ such

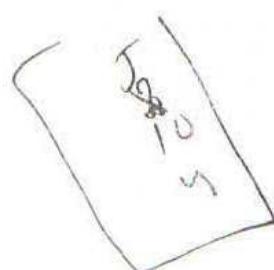
$$\text{such that } \|x_a\| = 1 \text{ and } \|x - x_a\| = a \text{ for all } x \in Y$$

- (b) Prove that dual of \mathbb{R}^n is \mathbb{R}^n



Q7. (a) Prove that every Hilbert space is reflexive

- (b) State and prove minimizing vector theorem



$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

fg

$$\begin{pmatrix} 6 \\ 1 \\ 6 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} 6 \\ 9 \\ 1 \\ 1 \end{pmatrix}$$

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