Roll No. ________________ **Subject: Math- V** **M.A/M.Sc: Part- I / Composite, 2^{nd} -A/10**

Sig of Dy. Superintendent. ________________

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University of Sargodha

M.A/M.Sc Part-1 / Composite, 2^{nd} -A/2010

Math- V Topology & Functional Analysis

Maximum Marks: 40 **Fictitious #:** ________

Time Allowed: 45 Min. **Signature of CSO:** ________

Objective Part

Note: **Cutting, Erasing, overwriting and use of Lead Pencil are strictly prohibited. Only first attempt will be considered.**

Q.1.A: Fill in the blanks. **(10)**

i. If A is a closed set then $F_r(A)$ ____________________.

ii. A subset A of a topological space X is said to be dense in X if $\overline{A}$ ____________________.

iii. Every open ball in a metric space is an ____________________.

iv. Every closed ball in a metric space X is a ____________________.

v. If $(X, \mathcal{J})$ is a door topology then every member of $\exists$ is both ____________________.

vi. For A and B subsets of a metric space (X, d). if $A \subseteq B$ implies $A^d \subseteq$ ____________________

vii. Any two norms defined on a finite dimensional linear space N are ____________________.

viii. For any subsets A and B of a topological space $(X, \mathcal{J})$ $\overline{A \cap B}$ ____________________ $\overline{A} \cap \overline{B}$.

ix. A complete normed space called ____________________ space.

x. A complete inner product space called ____________________ space.

B: Write true or false against each statement. **(10)**

i. In usual metric space R the set of integers have no limit point.

ii. ϕ and X are both open and closed sets in a metric space.

iii. In metric spaces a sequence may converge to two points.

iv. In normed space ℓ^{∞} is a Banach space.

v. In metric space every convergent sequence is a Cauchy Sequence.

vi. Every compact subset of a Hausdorff space is closed.

vii. A closed subspace of a compact space may not be compact.

viii. The real line R is not connected.

ix. A completely regular space may not be regular.

x. $A^{\perp}$ may not be a closed subspace of Hilbert Space H.

Q.2. Give short answer of the following: **(20)**

i. Discuss open ball in descrete metric space.

ii. In usual metric space R find A^o and $\overline{A}$ if $A =]2,5]$

iii. If $X = \{a,b,c\}$ write discrete topology on X.

iv. Discuss Cauchy Sequence in discrete metric space.

v. Show that the space Q of rationals as a subspace of real line R is not complete.

vi. Give an example of a metric space having two disjoint dense subsets.

vii. If A and B are subsets of a metric space X. then show that $A \subseteq B$ implies $A^d \subseteq B^d$.

viii. Let $\mathcal{J}$ be a topological space on N containing ϕ and all the subsets of the form $En = \{n, n+1, n+2, \_\_\_\_\_\}$, $n \in N$, list all the open sets containing positive integer 6.

ix. Show that the orthonormal set is linearly independent.

x. For complex inner product space V state the polarization identity.

University of Sargodha

M.A/M.Sc Part-1 / Composite, 2nd -A/2010

Math- V Topology & Functional Analysis

Maximum Marks: 60 **Time Allowed: 2:15 Hours**

Subjective Part

Note: **Attempt any three questions. All questions carry equal marks.**

Q.3. a. Let $(X, \mathcal{F})$ be a topological space and A be any arbitrary subset of X. then show that $\overline{A} = A \cup A^d$. (10)

b. Prove that the closed subspaces of Lindelof space are Lindelof. (10)

Q.4. a. Every subspace of a Tychonoff Space is a Tychonoff Space. (10)

b. Prove that every closed subspace of a compact space is compact. (10)

Q.5. a. Show that ℓ^∞ is a Banach Space. (10)

b. Prove that the continuous image of a connected space is connected. (10)

Q.6. a. Prove that any two norms on a finite dimensional linear space are equivalent. (10)

b. State and prove Riesz and Fischer theorem on Hilbert Spaces. (10)

Q.7. a. Let A be a proper complete subspace of an inner product space V then prove that $V = A \oplus A^\perp$ (10)

b. Let H be a Hilbert Space and f be any linear functional on H then there is a unique y in H such that $f(x) = \langle x, y \rangle$ for all x in H. (10)
