135

University of Sargodha

M.A/M.Sc Part- 1/Composite, 2nd -A/2009

Mathematics- V **Topology & Functional Analysis**

Maximum Marks: 60

Subjective Part

Time Allowed: 2:15 Ho

Note: **Attempt any three questions.**

Q.3. a. Prove: $f: X \to Y$ is continuous iff for any $A \subset X, f(\overline{A}) \subset \overline{f(A)}$ (1

b. Prove: A first countable space X is a Hausdorff space iff every convergent sequence in X has a unique limit. (1

Q.4. a. Define a sequentially compact space. Show that (0,1) as a subspace of R is not sequentially compact. (

b. Define a totally disconnected space. Show that the set Q of rational numbers is totally disconnected. (

c. Define a locally connected space. Let X and Y be locally connected. Then $X*Y$ is locally connected. (

Q.5. a. Prove: A metric space X is complete iff every nested sequence of non-empty closed sets whose diameters tend to zero has a non-empty intersection. (

b. State and prove Minkowski's inequality in normed spaces.

Q.6. a. The linear operator $T: R^n \to R^m$ is continuous defined by $Tx = Ax$, A is a real matrix (α_{jk}) with m rows and n columns. ((

b. Prove: A finite dimensional normed space is isometric to its second dual.

Q.7. a. Define an inner product space. Show that the space $C[a,b]$ is not an inner product space.

b. Define an orthonormal system. Show that an orthonormal system is linearly independent.

c. Let A and B be closed subspaces of a Hilbert space H such that $A \perp B$. Then $A+B$ is a closed subspace of H.