

University of Sargodha

M.A/M.Sc. Part-I/Composite, 1st A-Exam 2016

Mathematics: V Topology & Functional Analysis

Maximum Marks: 100

Time Allowed: 3 Hours

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the followings on your answer sheet:

(2*10)

- (i) Let A be the subset of a space X , then prove that $\text{Int}(A) \cup \text{Bd}(A) = \text{Cl}(A)$
- (ii) Define homeomorphism with an example.
- (iii) Define Normal space with an example.
- (iv) Prove that every T_1 -Space is T_0 -Space.
- (v) Define Compact space with an example.
- (vi) Define Euclidean metric.
- (vii) Define Cauchy-Schwarz inequality.
- (viii) Explain product of metric spaces.
- (ix) Define bounded linear functional.
- (x) What is Usual topology in \mathbb{R}^2 ?

Subjective Part (4*20)

- Q.2.** Prove that Separability is an open hereditary property.
- Q.3.** Prove that a completely regular space is also regular space.
- Q.4.** Let A be a subset of a topological space (X, τ) and let τ_A be the relative topology on A . Then A is τ -connected if and only if A is τ_A -connected.
- Q.5.** State and prove Minkowski inequality for sum.
- Q.6.** Prove that a subspace Y of a Banach space X is complete if and only if the set Y is closed in X .
- Q.7.** Prove that the space l^p with $p \neq 2$ is not an inner product space, hence not the Hilbert space.