Subject: Math: V **M.A/M.Sc: Part- I / Composite, 1st-A/2011**

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University of Sargodha

M.A/M.Sc Part-1 / Composite, 1st -A/2011

Math: V Topology & Functional Analysis

Maximum Marks: 40 **Fictitious #: ________**

Time Allowed: 45 Min. ***Objective Part*** **Signature of CSO: ________**

Note: **Cutting, Erasing, overwriting and use of Lead Pencil are strictly prohibited. Only first attempt will be considered.**

Q.1 (a)Fill in the blanks 10

(i) Every topological space (X,τ) ,where X is finite or τ consist of finite number of elements is called --------------------------------

(ii) An infinite set with co-finite topology is a ----------------- space.

(iii)Every convergent sequence will be ----------------- sequence.

(iv) Any number of open set in a topological space may not be ---------

(v) Every compact subset of a Hausdroff space is --------------------

(vi) The norm function $\|\bullet\|: N \to R$ is ------------------continuous.

(vii)The space l^p has the norm defined by ---------------------

(viii)The basis for the space C_0 is ------------------------------ BLANK

(ix)The norm function for the space of bounded linear space $B(X,Y)$ can be defined as ------------------, where X & y are normed spaces.

(x) If N is a normed space of finite dimension , then dimension of its conjugate space will be -----------------

(b) Determine whether the following statements true or false 10

(i)Every subspace of Banach space is a Banach space.

(ii) The concept of continuity and boundedness for a linear operator coincide

(iii) The space of rational number is not complete

(iv) If T is a linear operator, then R(T) is a vector space.

(v)The space R with usual topology is compact.

(vi) The real line R with usual topology is not a T_0 - space.

(vii) The indiscrete space is a regular space.

(viii) A completely regular space need not be regular space.

(ix)The dual space of R^n is R^n.

(x)Every metric space will be normed space.

Q.2 **Answer the following short questions.** 20

(i) If the range of a sequence $\{a_n\}$ is finite, then prove that it contains a convergent subsequence.

(ii)Let A be any subset of a discete topological space, then prove that it contains a convergent subsequence.

(iii)Define equivalent norms.

(iv)Show by giving a counter example that arbitrary intersection of open sets need not be open.

(v)Determine whether N is complete or not? Give reason for your answer.

(vi)If a linear operator is continuous on a point, then show that it is continuous on the whole space.

(vii)Show tat an orthonormal set is linearly independent.

(viii)What is n- dimensional unitary space?

(ix)Show that every normed space will be a metric space.

(x) Differentiate between dual space and second dual space.

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Math: V Topology & Functional Analysis

Maximum Marks: 60

Time Allowed: 2:15 Hou

Subjective Part

Note: **Attempt any three questions. All questions carry equal marks.**

Q.3

(a) Prove that every compact subset of a Hausdroff space is closed.

(b) Define Lindelof space and hence show that every second countable space is Lindelof.

Q.4

(a)Let (X,τ) be a topological space and $A \subseteq X$ .Let A^d and $\overline{A}$ denote the derived set and closure of A respectively then $\overline{A} = A \cup A^d$ .Also show that A is a closed subset of X iff $A \supset A^d$.

(b) Show that a bijective continuous function from compact space to a Hausdroff space is a homomorphism.

Q.5

(a) Show that any two norms of a finite dimensional space are equivalent.

(b) Prove that a subspace X of a real line R is connected if and only if X is an interval

Q. 6

(a) What is meant by Isometric Isomorphism ,Prove that dual space of l_1 is l^∞

(b) State and prove Cauchy-Schwartz inequality for inner product space.

Q.7

(a)Define the space of C [a, b] and hence show that C [a, b] is complete space.

(b) Does $d(x,y) = \int_a^b |x(t) - y(t)|dt$ define a metric or pseudometric on X if X is

(i)the set of all real valued continuous functions on [a, b].

(ii) The set of all real valued Riemann Integrable functions on [a, b]?

Give reasons for your answers.
