University of Sargodha

M.A/M.Sc Part-1 / Composite, 1st -A/2009

Mathematics- V Topology & Functional Analysis

Time Allowed: 2:15 Hours

Maximum Marks: 60

Subjective Part

Note: **Attempt any three questions.**

Q.3. (a) Let $(X, \mathcal{T})$ be a topological space and $A \subset X$, then $\overline{A} = A \cup A'$ (10)

(b) Define a normal space. Show that a metric space is a normal space. (10)

Q.4. (a) Define a Lebesgue number. Show that every sequentially compact metric space has a Lebesgue number. (10)

(b) Show that the Cartesian product of two connected spaces X and Y is connected. (5)

(c) Define a totally disconnected space. Show that a totally disconnected space is a Hausdorrf space. (5)

Q.5. (a) Define continuity in metric spaces. Show that $T:(X,d) \to (Y,d')$ is continuous iff the inverse image of every open subset of Y is open in X. (10)

(b) Prove: An absolutely convergent series in a normed space x is convergent if and only if X is a Banach Space. (10)

Q.6. (a) Define a bounded linear operator. Show that if a normed space X is finite dimensional, then every linear operator on x is bounded. (10)

(b) Define dual of a linear space N. Let N be an n-dimensional linear space, then its dual N^* also is n-dimensional. (10)

Q.7. (a) Let (e_k) be an orthonormal sequence in an inner product space X, then for every $x \in X$, $\sum_{k=1}^{\infty} |< x, e_k >|^2 \le \|x\|^2$. (10)

(b) Let A be a closed subspace of a Hilbert space H. Then $H = A \oplus A^{\perp}$ (10)