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University of Sargodha



M.A/M.Sc Part-I, 1st Annual Exam 2008

Mathematics-V

Topology & Functional Analysis

Maximum Marks: 40

Time Allowed: 45 Min.

Fictitious #: _____

Signature of CSO: _____

New Pattern

Objective Part

Note: Cutting, Erasing, overwriting and use of Lead Pencil are not allowed.

Q.1.A: Choose the correct option.

(5)

- The set of rational nos Q is _____.
a. Countable ✓ b. Uncountable
- Union of two topologies on a non-empty set _____.
a. is a topology ✓ b. May not be a topology
- The cofinite topology and the discrete topology on a finite set X _____.
a. Not comparable b. Coincide c. Not Coincide
- A complete normed space is called a _____.
a. Hilbert space b. Banach space ✓ c. Quotient space
- $\|x\|$ is _____.
a. The distance of x from the origin b. Absolute value of x .

B: Mark true or false.

(10)

- If T_1 and T_2 are topologies on X , Then $T_1 \cap T_2$ is also a topology on X . T F
- $X = (0,1)$ as a subspace of the real line with usual metric is complete. T F
- If a sequence (x_n) in (X, d) converges to a point x , then a subsequence (x_{n_k}) also converges to x . T F
- If A is closed in X , then the derived set of A may not be contained in A . T F
- $X = \{1,2\}$, then $\{1\}$ is open in (X, J) if J is a cofinite topology on X . T F
- A subspace Y of a complete metric space (X, d) is complete if Y is open in X . T F
- A subset C of a linear space N is convex if $\forall x, y \in C$ and $\alpha \in [0,1]$ $\alpha x + (1-\alpha)y \in C$. T F
- A linear functional $f : N \rightarrow F$ is said to be continuous at a point $x_0 \in N$, if given $\epsilon > 0, \exists \delta > 0$ such that $\forall x \in N$ $\|x - x_0\| < \delta \Rightarrow |f(x) - f(x_0)| < \epsilon$. T F
- Let V be an inner product space and $x \in V$, then $\langle x, x \rangle = 0$ T F
- Let $V(F)$ be an inner product space, then $x, y \in V$, and $a \in F \Rightarrow \langle x, ay \rangle = a \langle x, y \rangle$. T F

C: Fill in the blanks.

(5)

- If a Topological space has Cofinite Topology, then every infinite subset is _____.
- A subset A of a Topological space X is called dense in X if $\overline{A} = X$.
- The union of two disjoint open intervals on the real line is locally connected but not _____.
- A space in which every Cauchy sequence converges is called complete space.
- A linear functional f with domain $D(f)$ in a normed space is continuous iff " f " is _____.

P.T.

Give short answers.

(20)

Let X be a topological space and $A, B \subset X$, prove that $A \subset B \rightarrow \bar{A} \subset \bar{B}$.

State the Heine-Borel Theorem.

$S \subseteq \mathbb{R}$ is compact iff S is closed and bounded.

Show that a discrete metric space is complete.

Let X be a Cofinite topological space and $A \subset X$, find $\bar{A} = A$.

Let $X = \{a, b, c\}$. Show that $\beta = \{(a, b), (a, c)\}$ can't be a base for any topology on X .

Let X be a discrete space and Y any topological space, Then any $f: X \rightarrow Y$ is continuous.

Show that the real line with usual topology is separable.

Show that the real line is a normed space.

Define a linear operator. Show that the identity function $I: N \rightarrow N$ is a linear operator.

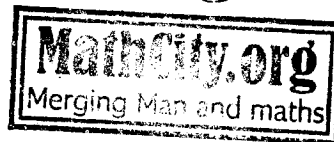
Let V be an inner product space and $x, y \in V$, if $x \perp y$, then prove $\|x + y\|^2 = \|x\|^2 + \|y\|^2$.

University of Sargodha

M.A/M.Sc Part-I, 1st Annual Exam 2008

Mathematics- V

Topology & Functional Analysis



Maximum Marks: 60

Time Allowed: 2:15 Hours

New Pattern

Subjective Part

Note: Attempt any three questions.

- Q.3. (a) Define relative topology. Let (Y, J_Y) be a subspace of (X, J) . show that (Y, J_Y) is a J_Y -closed if and only if $E = Y \cap F$, where F is a J -Closed subset of X . (10)
- (b) Define Product Topology. Let $\{X_i : i \in I\}$ be a collection of Hausdorff spaces and let $X = \prod_i X_i$ be the product space, then X is Hausdorff. (10)
- Q.4. (a) A closed Continuous image of a normal space is normal. (10)
- (b) A subspace X of the real line R is connected if and only if X is an interval. (10)
- Q.5. (a) Show that every countably Compact metric space is second countable. (10)
- (b) What is a complete metric space? Show that R^n is complete. (10)
- Q.6. (a) If a normed space X has the property that the closed unit ball $M = \{x : \|x\| \leq 1\}$ is compact, then X is finite dimensional. (10)
- (b) Let $T : D(T) \rightarrow Y$ be a linear operator, where $D(T) \subset X$; X, Y are normed spaces, then T is continuous iff T is bounded. (10)
- Q.7. (a) Prove that for any two elements x, y in an inner product space V ,
$$|\langle x, y \rangle| \leq \|x\| \|y\|$$
 (10)
- (b) State and prove minimizing vector Theorem. (10)