Subject: Mathematics (V) M.A/M.Sc: Part- I, I-A/08

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# University of Sargodha

## M.A/M.Sc Part-I, 1st Annual Exam 2008

**Mathematics-V Topology & Functional Analysis**

**Maximum Marks: 40** **Fictitious #:** ____________

**Time Allowed: 45 Min.** **Signature of CSO:** ____________

***New Pattern*** ***Objective Part***

**Note: Cutting, Erasing, overwriting and use of Lead Pencil are not allowed.**

**Q.1.A: Choose the correct option.** (5)

i. The set of rational nos Q is ____________.
a. Countable b. Uncountable

ii. Union of two topologies on a non-empty set ____________.
a. is a topology b. May not be a topology

iii. The cofinite topology and the discrete topology on a finite set X ____________.
a. Not comparable b. Coincide c. Not Coincide

iv. A complete normed space is called a ____________.
a. Hilbert space b. Banach space c. Quotient space

v. $\|x\|$ is ____________.
a. The distance of $x$ from the origin b. Absolute value of $x$.

**B: Mark true or false.** (10)

i. If $T_1$ and $T_2$ are topologies on $X$, Then $T_1 \cap T_2$ is also a topology on $X$. T F

ii. $X = (0,1)$ as a subspace of the real line with usual metric is complete. T F

iii. If a sequence $(x_n)$ in $(X, d)$ converges to a point $x$, then a subsequence $(x_{nk})$ also converges to $x$. T F

iv. If $A$ is closed in $X$, then the derived set of $A$ may not be contained in $A$. T F

v. $X = \{1,2\}$, then $\{1\}$ is open in $(X, J)$ if $J$ is a cofinite topology on $X$. T F

vi. A subspace $Y$ of a complete metric space $(X, d)$ is complete if $Y$ is open in $X$. T F

vii. A subset $C$ of a linear space $N$ is convex if $\forall x, y \in C$ and $\alpha \in [0,1]$ $\alpha x + \alpha y \in C$. T F

viii. A linear functional $f : N \to F$ is said to be continuous at a point $x_0 \in N$, if given $\varepsilon > 0, \exists$ $a\delta > 0$ such that $\forall x \in N \|x - x_0\| < \varepsilon \Rightarrow |f(x) - f(x_0)| < \delta$. T F

ix. Let $V$ be an inner product space and $x \in V$, then $<x, x> = 0$ T F

x. Let $V(F)$ be an inner product space, then $x, y \in V$, and $a \in F \Rightarrow <x, ay> = a <x, y>$. T F

**C: Fill in the blanks.** (5)

i. If a Topological space has Cofinite Topology, then every infinite subset is ________________.

ii. A subset $A$ of a Topological space $X$ is called dense in $X$ if ____ A = X ____.

iii. The union of two disjoint open intervals on the real line is locally connected but not ________________.

iv. A space in which every Cauchy sequence converges is called ____ complete ____.

v. A linear functional $f$ with domain $D(f)$ in a normed space is continuous iff "$f$" is ________________.

P.T.O

Give short answers. (20)

Let $X$ be a topological space and $A, B \subset X$, prove that $A \subset B \rightarrow \bar{A} \subset \bar{B}$.

State the Heine-Borel Theorem.

$S \subseteq \mathbb{R}$ is compact iff $S$ is bounded and closed

Show that a discrete metric space is complete.

Let X be a Cofinite topological space and $A \subset X$, find $\bar{A}$ $\subseteq A$

A

Let $X = \{a, b, c\}$. Show that $\beta = \{(a,b),(a,c)\}$ can't be a base for any topology on $X$.

Let $X$ be a discrete space and Y any topological space, Then any $f : X \rightarrow Y$ is continuous.

Show that the real line with usual topology is separable.

Show that the real line is a normed space.

Define a linear operator. Show that the identity function $I : N \rightarrow N$ is a linear operator.

Let $V$ be an inner product space and $x, y \in V$, if $x \perp y$, then prove $\|x + y\|^2 = \|x\|^2 + \|y\|^2$.

(111)



# University of Sargodha

M.A/M.Sc Part-I, 1st Annual Exam 2008

Mathematics- V Topology & Functional Analysis

Maximum Marks: 60 Time Allowed: 2:15 Hours

*New Pattern* *Subjective Part*

Note: Attempt any three questions.

Q.3. (a) Define relative topology. Let $(Y, J_y)$ be a subspace of $(X, J)$. show that $E \subset Y$ is a $J_y$-closed if and only if $E = Y \cap F$, where F is a $J$-Closed subset of $X$. (10)

(b) Define Product Topology. Let $\{X_i : i \in I\}$ be a collection of Hausdorff spaces and let $X = \pi_i X_i$ be the product space, then $X$ is Hausdorff. (10)

Q.4. (a) A closed Continuous image of a normal space is normal. (10)

(b) A subspace $X$ of the real line $R$ is connected if and only if $X$ is an interval. (10)

Q.5. (a) Show that every countably Compact metric space is second countable. (10)

(b) What is a complete metric space? Show that $R^n$ is complete. (10)

Q.6. (a) If a normed space $X$ has the property that the closed unit ball $M = \{x : \|x\| \le 1\}$ is compact, then $X$ is finite dimensional. (10)

(b) Let $T : D(T) \to Y$ be a linear operator, where $D(T) \subset x$; $X$, $Y$ are normed spaces, then $T$ is continuous iff $T$ is bounded. (10)

Q.7. (a) Prove that for any two elements $x$, $y$ in an inner product space $V$, (10)

$$|< x, y >| \le \|x\|.\|y\|$$

(b) State and prove minimizing vector Theorem. (10)

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