Subject: Math: I/VI   M.A/M.Sc: Part- II/Composite, 2nd -A/10

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University of Sargodha

M.A/M.Sc Part-II/Composite, 2nd -A/2010

Math: I/VI   **Advance Analysis**

Maximum Marks: 40   **Fictitious #:** ____________

Time Allowed: 45 Min.   **Signature of CSO:** ____________

Objective Part

Note: **Cutting, Erasing, overwriting and use of Lead Pencil are strictly prohibited. Only first attempt will be considered.**

Q.1.A: Write true or false. (10)

i. A well ordered set need not satisfy the descending chain condition.

ii. Suppose $f: A \to B$ & $g: A \to B$ are two order isomorphism then $f = g$.

iii. A well ordered set is order isomorphic to one of its initial segments.

iv. A set E equipped with the relation "a divides b" is totally ordered.

v. An ideal in a well ordered set C can not be all of C.

vi. If E is a null set in a measure space (X, A, μ) then every subset of E is also a null set.

vii. Limit of a monotone sequence of sets always exists.

viii. Product of two simple functions is a simple function.

ix. Countable union of countable sets is countable.

x. For two sets A and B $A \times B = B \times A$

B: Fill in the Blanks. (10)

i. If $\{E_n\}_1^\infty$ is an increasing sequence of subsets of a set X then $\lim_{n\to\infty} E_n =$ ________________.

ii. If A and B are two sets in a measure space (X, A, μ) with $A \subseteq B$ then $\mu(A) \le \mu(B)$ this property is called ________________.

iii. Let μ^* be an outer measure on a set X. then a subset E of X is μ^* measurable if $\mu^*(A) = \mu^*(A \cap B) +$ ________________ holds for every A in $P(X)$.

iv. Let D be collection of all open sets in a topological space X, then the τ-algebra generated by D is called ________________.

v. If ϕ is a simple function and $\{a_1, a_2 \ldots\ldots\ldots\ldots a_n\}$ is the set of non-zero values of ϕ then the canonical representation of ϕ is given by ________________.

vi. Let μ^* be an outer measure on a set X. If $E, F \in P(X)$ and $\mu^*(F) = 0$ then $\mu^*(E \cup F) =$ ________________

vii. If $\{En\}_1^\infty$ is a decreasing sequence of subsets of a set X. then $\lim_{n\to\infty} E_n =$ ________________

viii. For a disjoint sequence $\{E_n\}_1^\infty$ in a measure space (X, A, μ) we have $\mu\left(\bigcup_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} \mu(E_n)$. this property is called ________________.

University of Sargodha

M.A/M.Sc Part-II/Composite, 2nd -A/2010

Math: I/VI **Advance Analysis**

Maximum Marks: 60 **Time Allowed: 2:15 Hours**

Subjective Part

Note: **Attempt any three questions. All questions carry equal marks.**

Q.3. a. Given a measure space $(X, \mathcal{A}, \mu)$. Let ϕ be a non-negative simple function on X. prove that the function $V(A) = \int_A \phi d\mu$ for $A \in \mathcal{A}$ is a measure. (10)

b. Show that the function (10)

$$f(x) = \begin{cases} 0, & x \text{ is rational} \quad x \in [0,1] \\ 1, & x \text{ is irrational} \quad x \in [0,1] \end{cases}$$

is not Riemann integrable but it is Lebesgue integrable.

Q.4. a. Let μ^* be an outer measure on a set X and let $m(\mu^*)$ be the collection of all μ^* measurable subsets of X. Prove that μ^* when restricted to $m(\mu^*)$ is a measure on $m(\mu^*)$. (12)

b. Let $(X, \mathcal{A})$ be a measurable space and let $E \in P(X)$ then prove that the characteristic function χ_E on X is $\mathcal{A}$ measurable function iff $E \in \mathcal{A}$ (8)

Q.5. a. Show that $J_{1/2}(x) = \left(\frac{2}{\pi x}\right)^{1/2} \sin x.$ (10)

b. Prove that $x^2 J_n''(x) = n(n-1)J_n(x) - (2n+1)xJ_{n+1}(x) + x^2 J_{n+2}(x)$ (10)

Q.6. a. Let $(X, \leq)$ & $(X^1, \leq')$ be two well ordered sets. If X is order isomorphic to X^1. then prove that there exists only one order isomorphism between X and X^1. (10)

b. Prove that the interval [0, 1] is not countable. (10)

Q.7. a. Use series solution Frobensus method to solve the following differential equation $x(x-1)y'' + (3x-1)y' + y = 0$ (12)

b. Show that the function $F(x,t) = \frac{1}{\sqrt{1-2xt+t^2}}$ generates polynomial $P_n(x)$ (8)

then show that $\frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} P_n(x)t^n$