University of Sargodha

M.A/M.Sc Part- II/Composite, 1st-A/2013

Mathematics: I/VI **Advance Analysis**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

Objective Part

Q.1. Write short answers of the following in 2 or 3 lines only on your answer sheet.

2×10

(i) Suppose that $A \approx B$ and $C \approx D$, then show that $A \times C \approx B \times D$.

(ii) Let $\{E_n\}_1^\infty$ be a sequence in a σ-algebra $\mathcal{A}$ of subsets of a set X. Let $\{F_n\}_1^\infty$ be the sequence defined by $F_1 = E_1, F_n = E_n \setminus \bigcup\limits_1^{n-1} E_i$ for $n \geq 2$. Show that $\bigcup\limits_1^\infty E_n = \bigcup\limits_1^\infty F_n$.

(iii) Given any two sets x and y, show that there is a unique set z whose elements are x and y.

(iv) Let X be a nonempty set and μ^* be an outer measure on $P(X)$. If $\mu^*(E) = 0$ for some $E \in P(X)$, then show that every subset E_0 of E is μ^*-measurable.

(v) Show that the Lebesgue measure space $(\mathbb{R}, \mathfrak{M}_L, \mu_L)$ is σ-finite measure space.

(vi) Show that the converse of the statement "every step function is a simple function" is not true.

(vii) Show that if $f = 0$ a.e. on a set $D \in \mathcal{A}$ in a measure space $(X, \mathcal{A}, \mu)$, then $\int\limits_D f d\mu = 0$.

(viii) Prove that the singletons are null sets in the Lebesgue measure space $(\mathbb{R}, \mathcal{M}_L, \mu_L)$.

(ix) Show that $\int\limits_0^\infty e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$.

(x) State Zorn's Lemma.

(Subjective Part)

Q: 2	(a) Given a measure space $(X, \mathcal{A}, \mu)$. Let f_1 and f_2 be bounded real valued $\mathcal{A}$-measurable functions on a set $D \in \mathcal{A}$ with $\mu(D) < \infty$. If $f_1 = f_2$ a.e. on D, then $\int_D f_1 d\mu = \int_D f_2 d\mu$.	10
	(b) Let G be a group, then assuming Zorn's lemma, prove that G has a maximal abelian subgroup.	10
Q.3	(a) Prove that $\mu_L^*(I) = l(I)$, where I is a finite closed interval.	10
	(b) Prove that the function $\exp(\frac{x}{2}(t - \frac{1}{t}))$ generate the Bessel functions.	10
Q.4	(a) Given a measure space $(X, \mathcal{A}, \mu)$. Let $\{f_n\}_1^\infty$ be a sequence of extended real valued $\mathcal{A}$-measurable functions on a set $D \in \mathcal{A}$ and $f = \lim\limits_{n\to\infty} f_n$. Then prove that $\lim\limits_{n\to\infty} \int_D f_n d\mu = \int_D f d\mu$.	10
	(b) Prove that $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$.	10
Q. 5	(a)Show that if f and g are $\mathcal{A}$-measurable functions on a set $D \in \mathcal{A}$ in a measure space $(X, \mathcal{A}, \mu)$. Then prove that $f+g$ and $\frac{f}{g}$, $g \neq 0$ are $\mathcal{A}$-measurable.	10
	(b)Solve Bessel's equation by Frobenius method.	10
Q. 6	(a) Given a measure space $(X, \mathcal{A}, \mu)$. Let f, f_1, f_2 be bounded real valued $\mathcal{A}$-measurable functions on a set $D \in \mathcal{A}$ with $\mu(D) < \infty$. Then prove that • $\int\limits_D cf d\mu = c\int\limits_D f d\mu$, where c is a constant. • $\int\limits_D (f_1 + f_2) d\mu = \int\limits_D f_1 d\mu + \int\limits_D f_2 d\mu$	14
	(b)Prove that the interval $[0,1]$ is uncountable.	06
Q. 7	(a) Given a measure space $(X, \mathcal{A}, \mu)$. Let f be a bounded real valued $\mathcal{A}$-measurable function on a set $D \in \mathcal{A}$ in a measure space $(X, \mathcal{A}, \mu)$. If $f \geq 0$ a.e. on D and $\int\limits_D f d\mu = 0$, then prove that $f = 0$ a.e. on D.	12
	(b)Evaluate the integral $\int\limits_{-1}^{1} P_n(x) dx$.	08