Roll No. ________________ Subject: Math: II/VII M.A/M.Sc: Part- II / Composite, 1st -A/2011

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University of Sargodha

M.A/M.Sc Part-II / Composite, 1st -A/2011

Math: II/VII Advance Analysis

Maximum Marks: 40 Fictitious #: ______

Time Allowed: 45 Min. Signature of CSO: ______

Objective Part

Note: Cutting, Erasing, overwriting and use of Lead Pencil are strictly prohibited. Only first attempt will be considered.

Q.1 A. Write true or False

i. For a constant function the Lebesgue integral and Reimann integral coincides.

ii. Every Borel set is m^*-mble

iii. Every simple function defined on X partitions X.

iv. Limit of monotone sequence of sets always exists.

v. Countable union of countable set need not be countable

vi. A set is said to be denumerable if it is equivalent to $\mathbb{R}$

vii. For two set A and B $A * B = B * A$

viii. Every simple function is a step function

ix. Every simple function is Lebesgue integrable

x. Countable union of countable set is countable.

Fill in the Blanks (10)

i. The integral representation of

$$\frac{\sqrt{x}\,\sqrt{y}}{2\sqrt{x+y}} = \ldots$$

ii. The Legender Polynomials can be generated by the function ....

iii. $\int_{-1}^{1} P_n^2(x)\,dx = \ldots$

iv. If there exists an injective function $f: A \to B$ then cardinality of $A \leq \ldots$

v. A step function φ is measureable if ....

vi. A measureable function f defined on E is integrable if .... integrable

vii. Let $\{f_n\}$ be a sequence of non-negative measureable function with $f_n \to f$ on E then

$$\lim\inf \int f_n = \ldots$$

viii. $R\int_a^b f(x)\,dx = \ldots$

ix. $\varphi = \sum_{i=1}^{n} a_i \chi_{A_i}$ is called canonical if all a_is are ....

x. $m(\cup_i E_i) = \sum_i m(E_i)$ if E_is ....

(P.T.O)

Q2 SHORT Questions. 5×2

i Let f and g outer measure Show that $f+g$ is outer measure

ii Show that Lebesgue measure of a Singleton set is zero

iii If E_1 and E_2 are m^*-mble and $E_1 \subseteq E_2$ then

$$m(E_2 | E_1) = m(E_2) - m(E_1)$$

iv If E is m^*-mble So is $(E+x)$.

For syllabus and old papers, please visit http://www.MathCity.org

University of Sargodha

M.A/M.Sc Part- II/Composite, 1st -A/2011

Math: II/VII **Advance Analysis**

Time Allowed: 2:15 Hours

Maximum Marks: 60

Subjective Part

Note: Attempt any three questions. All questions carry equal marks.

Q3(a)	Let g be integrable on E and $\{f_n\}$ be sequence of measureable functions such that $\lvert f_n\rvert \le g$ on E. Suppose that $\lim_n f_n = f$ a.e on E then $$\int_E \lim_n f_n = \int_E f = \lim_n \int_E f_n.$$	10+10
b.	Let X be a set of ordinals then prove that $\cup X$ is an ordinal and the least upper bound of X.	
Q4a.	Let $E_1, E_2, E_3 \ldots$ be mutually disjoint m^*-mble set then $\cup_i E_i$ is also m^*-mble	10+10
b.	Prove that a chain is well ordered iff it does not contain an infinite descending sequence	

Q5a. Let f and g be non-negative measureable function on E then (8)

$$\int_E cf = c\int_E f$$

b. Prove the following Equality. 12

$$\int_0^{\pi/2} \cos^{2x-1}\theta \sin^{2y-1}\theta \, d\theta = \frac{\sqrt{x}\,\sqrt{y}}{2\sqrt{x+y}}$$

Q6a. Show that 10+10

$$J_{1/2}(x) = \left(\frac{2}{\pi x}\right)^{1/2} \sin x.$$

b. If $\{E_i\}$ is a sequence of m^*-mble and $E_1 \subseteq E_2 \subseteq E_3 \subseteq \ldots$ then

$$m(\cup_i E_i) = \lim_{n\to\infty} m(E_n).$$

Q7a. Find $P_4(x)$ using any suitable method.

b. Let f and g be non-negative measureable function on E then

$$\int_E f+g = \int_E f + \int_E g.$$