University of Sargodha

Maximum Marks: 60   **Time Allowed: 2:15 Hours**

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Subjective Part

Note: **Attempt any three questions. All questions carry equal marks.**

Q.3. a.   Find $P_u(x)$ using any suitable method.   (10)

b.   Let μ^* be an outer measure on a set X. if $E_1, E_2 \in P(X)$ are μ^* measurable then prove   (10)
that $E_1 \cup E_2$ is μ^* measurable.

Q.4. a.   Let $(\Re, m_L, \mu_L)$ be a Lebesgue measure space show that every countbale subset of R   (8)
is a null set in $(\Re, m_L, \mu_L)$

b.   Let μ be a measure on a T algebra A of subset of a set X. If $\{E_n\}_1^\infty$ is a decreasing   (12)
sequence in A then prove that $\underset{n \to \infty}{Lim}\, \mu(E_n) = \mu\left(\underset{n \to \infty}{Lim}\, E_n\right)$

Provided that $\mu(E_1) < \infty$

Q.5. a.   Prove that a chain is well ordered iff it does not contain an infinite descending   (10)
sequence.

b.   Let (X, A) be a measurable space and let $E \in P(X)$ then prove that the characteristic   (10)
function χ_E. On X is A measurable function iff $E \in A$.

Q.6. a.   Let X be a set of ordinals then prove that UX is an ordinal and the least upper bound   (10)
of X.

b.   Prove the following equality.   (10)

$$\int_0^{\frac{\pi}{2}} \cos\theta^{2x-1}\, \sin\theta^{2y-1}\, d\theta = \frac{\sqrt{x}\sqrt{y}}{2\sqrt{x+y}}$$

Q.7. a.   Show that $J_{\frac{1}{2}}(x) = \left(\frac{2}{\pi x}\right)^{\frac{1}{2}} \sin x$ (10)

b.   Prove that   (10)

$$x^2 J_n''(x) = n(n-1)\, J_n(x) - (2n+1)x\, J_{n+1}(x) + x^2 J_{n+2}(x)$$

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