

Subjective Part

Note: **Attempt any three questions. All questions carry equal marks.**

- Q.3.** a. Solve the integral equation (10)

$$U(x) = f(x) + \lambda \int_0^{2\pi} (\sin x \sin y) U(y) dy$$

- b. State and prove lagrange identity. (10)

- Q.4.** a. Derive Euler-langrange equation for two independent variables. (10)

- b. Find the shortest distance between points $A(1, -1, 0)$ and $B(2, 1, -1)$ in the plane $15x - 7y + z - 22 = 0$ (10)

- Q.5.** a. If $U_m(x)$ and $U_n(x)$ are the solutions of a regular SL-System corresponding to distinct eigen values λ_m and λ_n respectively, then $U_m(x)$ and $U_n(x)$ are orthogonal w.r.t weight function $\gamma(x)$ i.e. (10)

$$\int_a^b U_m(x) U_n(x) \gamma(x) dx = 0 \quad m \neq n$$

- b. Find the Fourier transform of the following (10)

$$f(x) = \begin{cases} \sin x & 0 \leq x \leq \pi \\ 0 & x > \pi \end{cases}$$

- Q.6.** a. Construct Green's function for $\frac{d}{dx} \{ (1-x^2) U' \} - \frac{h^2}{1-x^2} U + \lambda r(x) U = 0$ (10)

- b. Obtain the general method for obtaining the solution of the Fredholm integral equation of second kind when kernel is separable. (10)

- Q.7.** a. Find the Fourier transform of the Gaussian function $g(x) = N e^{-\alpha x^2}$ where N and α are constants, $\alpha > 0$. (10)

- b. Use the inversion integral (or residue) method to calculate the laplace transform of $F(S) = \frac{1}{S^2(S+1)}$. (10)