University of Sargodha

M.A/M.Sc Part- II/Composite, 2nd-A 2009

Mathematics: II **Methods of Mathematical Physics**

Maximum Marks: 60 ***Subjective Part*** **Time Allowed: 2:15 Hours**

Note: **Attempt any three questions.**

Q2 a) Discuss the nature of Partial Differential equation
$(1-x^2)u_{xx} - 2xyu_{xy} + (1-y^2)u_{yy} + 2u_x + 3u_y - 2u = 0$ and draw its figure. 10

b) Find the Fourier series for the function $f(t) = \begin{cases} -\pi & -\pi \le t < 0 \\ x & 0 < t \le \pi \end{cases}$ 10

Q3 a) Solve the non-Homogeneous initial/boundary value problem for Steady State Solution only. 10

$$\frac{\partial_2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t} + \frac{T_1}{P^2}. \qquad 0 \prec x \prec a \ , t \succ 0$$

$$u(0,t) = T, \quad \frac{\partial u(a,t)}{\partial x} = 0, \ \ t \succ 0$$

$$u(x,0) = T, \qquad 0 \prec x \prec a$$

b) Let w (x, t) be the solution of the equation 10

$$\frac{\partial_2 w}{\partial x^2} = \frac{1}{c^2}\frac{\partial_2 w}{\partial t^2} \qquad 0 \prec x \prec 2a \ , t \succ 0$$

$$w(0,t) = 0 = w(a,t) \qquad t \succ 0$$

$$w(x,0) = p(x) \qquad \frac{\partial w(x,0)}{\partial t} = 0 \qquad 0 \prec x \prec 2a$$

Where p(x) is a function whose graph is a isosceles triangle of width '2a' and height '$\frac{h}{2}$' Find w (x, t) for x=0.5a and $t = \frac{0.8a}{c}$.

Q4 a) If W preserve the same sign thought out the interval [a,b], then show that the eigen value of SL-system 10

$$\frac{d}{dx}\left[p\frac{du}{dx}\right] + q(x)u + w\lambda u = 0$$

$$\alpha_1 u(a) + \alpha_2 u'(a) = 0$$

$$\beta_1 u(b) + \beta_2 u'(b) = 0$$

is real

b) Construct the green function of the following system. 10

$$\frac{d_2 u}{dx^2} - \frac{4x}{2x-1}\frac{du}{dx} + \frac{4}{2x-1}u + \lambda r(x)u = 0$$

u(0) =0 & u(1)=0

Q5 a) If f(x) is a real function defined over $(-\infty, +\infty)$ and the integral $\int_{-\infty}^{+\infty} f(x)dx$ is absolutely convergent ,then $f(x) = \frac{1}{\pi}\int_0^\infty dk \int_{-\infty}^{\infty} \mathrm{Cos}\, k(\xi - x) f(\xi) d\xi$ 10

P.T.O

b) Show That

i) $\ell^{-1}\left\{\frac{F(s)}{s^2}\right\} = \int_0^t\int_0^\tau f(\lambda)d\lambda d\tau$

ii) $\ell^{-1}\left\{Tan^{-1}(\frac{1}{s})\right\} = \frac{1}{t}Sin t$

iii) $\ell^{-1}\left\{t\int_0^t \tau e^{-\tau}d\tau\right\} = \frac{3s+1}{s^2(s+1)^3}$

Q.6 a) State & prove "EULER LAGRANGE EQUATION" of variation.

b) Find the Euler Lagrange equation of the following functions

i) $F = x^2y^2 - y'^2$

ii) $F = \sqrt{xy} + y'^2$

iii) $F = Sin(xy')$

iv) $F = \frac{x^2y'}{\sqrt{1+y'^2}}$