University of Sargodha

M.A/M.Sc Part- II/Composite, 1st-A/2014

Mathematics: VII **Methods of Mathematical Physics**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective

Q. 1. Give the short answers of the followings:

i). State principle of superposition, ii). State existence theorem

iii). Define eigenvalue problem iv). Define orthogonal functions.

v). State Lagrange's identity vi). Define Laplace transform

vii). Define pole of a function of order *n* also give special example.

viii). Show that $L\{f'(t)\} = sF(s) - f(0)$

ix). Define a functional x). Define homogeneous Fredhalm integral equation

SUBJECTIVE

Q 2. a). Solve $\frac{d^2x}{dt^2} + \lambda x = 0$ for all possible values of the parameter λ with the boundary conditions $x(0) = 0,\ x(a) = 0$. (10)

b). Determine eigenvalues and eignfunctions of the system $\frac{d^2u}{dt^2} + (\lambda + 1)u = 0$ with the boundary conditions, $x(0) = 0,\ x(\pi) = 0$. (10)

Q 3. a). Prove that, for Laplace transform, the following relation holds:

$$L\{f^n(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f(0) -, \ldots, f^{n-1}(0)$$ (10)

b). Find Laplace transform of Bessel function of first kind of order zero, i.e.,

$$J_0(t) = \frac{1}{\pi}\int_0^{\pi} \cos(t\sin\theta)\,d\theta. \qquad (10)$$

Q 4. a). Find the Fourier transform of $f(x) = \frac{a}{x^2 + a^2}$ for all possible values of k. (10)

b). Find the Fourier sine transform of $f(x) = e^{-x}\cos x$ (10)

Q 5. (a). Construct Green's function associated with the problem $u'' + \lambda u = 0,$ with bc's $u(0)=0, u(1)=0$. (10)

b). Solve the problem by using Green's function

$$\frac{du^2}{dx^2} = f(x) \quad \text{with bc's} \quad u(0)=\alpha, u(l)=\beta. \qquad (10)$$

Q 6. a). Define stationary value and find the extemal curve of the functional *J*, given by

$$J = \int_A^B y'(1 + x^2 y')\,dx,$$

which passing through A(1,3) and B(2,5). (10)

b). State and prove fundamental theorem of calculus of variations (one independent variable). (10)

Q 7. a). Solve the Fredhalm integral equation, given by

$$\phi(s) = s + \lambda\int_0^1 (st^2 + s^2 t)\phi(t)\,dt. \qquad (10)$$

b). Obtain the general method for obtaining the solution of the Fredholm I.E. of second kind when kernel is separable. (10)