University of Sargodha

M.A/M.Sc Part-II / Composite, 1st -A/2011

Math: II/VII Method of Mathematical Physics

Maximum Marks: 40 **Fictitious #:** _______

Time Allowed: 45 Min. ***Objective Part*** **Signature of CSO:** _______

Note: **Cutting, Erasing, overwriting and use of Lead Pencil are strictly prohibited. Only first attempt will be considered.**

Q.1 (A) Fill in the blanks (5x1=5)

(i) $\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ is known as ----

(ii) $\frac{d}{dx}\left\{(1-x^2)\frac{du}{dx}\right\} + (n(n-1))u = 0$ is called ---- differential Equation

(iii) $L\left\{\frac{e^{-2s}}{s^3}\right\}$ ----

(iv) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$ is called ----

v) $\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y} = 2u$ is called ----

(B) Encircle the correct answer (5x1=5)

(i) the PDE
$a(x,y)\,u_{xx} + b(x,y)\,u_{xy} + c(x,y)\,u_{yy} = f(x,y,u,\ldots)$
is elliptic if

-) $b^2-4ac<0$ -) $b^2-4ac>0$ -) $b^2-4ac=0$
-) $b^2-ac<0$

(ii) $y\frac{dy}{dx} + x^2 = 1$ is ---- D.E

-) linear of order 1 -) non-linear of order 2
-) linear of order 2 -) non-linear of order

(P.T.O

(iv) SL-Equation is named after the two ___ mathematicians

- German & French
- Greek & French
- German & Greek
- Greek & American

(v) $\nabla^2 u + k^2 u = 0$ is called

- Poisson Equation
- Heat Equation
- Helmholtz Equation
- Laplacian operator

(v) $F\{f(x-a)\} =$

- $e^{-ika} F(k)$
- $e^{ika} F(k)$
- $e^{-ika} F(x)$
- $e^{ika} F(x)$

C) Tick True or False (10)

(i) A differential Equation which always possesses the trivial solution $u=0$ is called non-homogenous D.E (T/F)

(ii) $\frac{\partial^2 u}{\partial x^2} - u\frac{\partial u}{\partial y} = 1$ is linear and homogenous PDE. (T/F)

(iii) the D.E $xy'' + (1-x)y' + \lambda y = 0$ is called Hermite D.E. (T/F)

(iv) $\frac{d^2u}{dx^2} + \lambda u = 0$ is an example of periodic S-L system (T/F)

(v) $\int_0^\infty e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$ (T/F)

(vi) $\Gamma(n+1) = (n+1)\Gamma(n)$ (T/F)

ii)	$L^{-1}\{1/s\} = t$	(T/F)
iii)	the D.E $x(x-3)y'' + xy' + y = 0$ has no singular point	(T/F)
ix)	$U(0) = c_1$, $U(b) = c_2$ are Neumann boundary conditions.	(T/F)
x)	the exponential order of $\sin at$ is 1	(T/F)

Q. No. 2 Give short answers of the followings (10x2=20)

(i) Define Regular S-L System

(ii) Define Hypergeometric differential Equation.

(iii) what is the order and degree of
$$2\left(\frac{d^2}{dx^2}(xy)\right)^3 - 7\left(\frac{dy}{dx}\right)^5 + y = 0$$

(iv) State Lagrange identity.

v) Write Volterra integral Equation of second kind.

vi) Define Geodesic.

vii) show that
$$L\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

viii) Define unit step function.

ix) what is Dirac delta function.

x) Define Dido's problem.

(P.T.O)

M.A/M.Sc Part- II/Composite

Math: II/VII **Method of Mathematical Physics**

Maximum Marks: 60 **Time Allowed: 2:15 Hours**

Subjective Part

Note: **Attempt any three questions. All questions carry equal marks.**

Q.3 (a) If $f(x)$ is piecewise smooth and absolutely integrable on an $(-\infty, \infty)$, then its Fourier transform is bounded and continuous. (10)

b) Solve $(e^t x')' + e^t \lambda x = 0$ with $x(0) = 0$ & $x(a) = 0$ where prime denotes the derivative w.r.t t. (10)

Q.4 (a) Evaluate the Resolvent Kernal of the integral Equation (10)

$$g(s) = f(s) + \lambda\int_0^1 (s+t)\, g(t)\, dt$$

b) State and prove Heat Equation in one dimension (10)

Q.No 5 (a) State and prove Lagrange identity (10)

(b) Evaluate (i) $L\{\sin^2 t\}$ (ii) $\int_{-\infty}^{\infty} e^{-t^2} dt$ (10)

Q.No 6 (a) Use calculation of variations to prove that a straight line is the shortest distance between two points in a plane (10)

(b) Derive Euler's-Langrange Equation Also discuss its special cases (10)

Q.No 7 (a) (i) State and prove shifting property of Fourier transform (10)

(P.T.O)

Question No.	Questions	Marks
(ii)	State and prove convolution theorem for 2-dim case.	(10)
b)	Eigensolutions corresponding to distinct eigenvalues of a periodic S-L system are orthogonal w.r.t. the weight function $r(x)$.	