

Maximum Marks: 60

Time Allowed: 2:15 Hours

Subjective Part

Note: Attempt any three questions. All questions carry equal marks.

- Q.2 a) Given the following Ordinary / Partial differential equations, state which of them are linear/non linear and Homogeneous /non Homogeneous. 10

i) $u_{xx} - \frac{1}{\alpha^2} u_t = 0$

ii) $5y'' + xy' + 7y = x^2 + 7x$

iii) $u'' + p(x)u' + q(x)u = 0$

iv) $u'' + 3u' + 2u = e^x$

v) $u' + x \cos u = \sin x$

- b) Find the Fourier series for the function $f(t) = \begin{cases} 0 & -\pi \leq t < 0 \\ \frac{t}{\pi} & 0 < t \leq \pi \end{cases}$ 10
 with $f(t+2\pi) = f(t)$

- Q.3 a) A string of length $\frac{a}{2}$ is stretched between two fixed points and is made to vibrate by plucking it at the middle point through a short distance $\frac{h}{4}$. Formulate and Solve the problem. 10

- b) Drive one-dimensional heat equation. 10

- Q.4 a) For SL-System $\frac{d^2u}{dx^2} + \lambda u(x) = 0 \quad 0 \leq x \leq l$ (4+4+2)
 $u'(0) = 0 = u'(l)$

Verify the following results

- iv. There are infinite numbers of eigen values with a smallest but not largest.
- v. The nth eigen function has exactly (n-1) zeroes.
- vi. The eigen functions are orthogonal.

- b) Construct the green function of the following system 10

$$\left[(1-x^2)u' \right]' - \frac{h^2}{1-x^2} u + \lambda r(x)u = 0$$

$$u(1) = \text{finite}$$

$$u(-1) = \text{finite}$$

- Q.5 a) Show that $F_s \{ f'(x) \} = k \sqrt{\frac{2}{\pi}} f(0) - k^2 F_s(k)$ if $f(x)$ is real, $|f(x)| \rightarrow 0$ and $|f'(x)| \rightarrow 0$ as $x \rightarrow \infty$ 10

- b) To Solve the Differential Equation by using Laplace Transformation 10

$$y'' + 4y' + 6y = 1 + e^{-t}$$

$$y(0) = 0 \quad y'(0) = 0$$

- Q.6 a) Find the equation of the path in space down which a particle will fall from One point to another point in the shortest possible time. 10