University of Sargodha

M.A/M.Sc Part- II/Composite, 1st-A/2014

Mathematics: VIII Numerical Analysis

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: Objective part is compulsory. Attempt five questions from subjective part by selecting two questions from section-I, two questions from section-II and one questions from section-III.

Objective Part

Q.1. Give short answers to the following questions. Cutting, overwriting, erasing and lead pencil are strictly prohibited. Only first attempt will be considered. (10*2)

(i) If Δ, ∇ and δ denotes the forward, backward and central differences then show that

$$\mu\delta = \frac{\nabla + \Delta}{2}$$

(ii) Construct the forward difference table for the given data

x	0.1	0.3	0.5	0.7	0.9	1.1	1.3
y	0.003	0.067	0.148	0.248	0.370	0.518	0.697

(iii) What do you mean by inverse interpolation, what are the limitations for using Newton's interpolation method and write down the formula for Newton's forward difference interpolation method?

(iv) Write down the types of errors and discuss convergence criteria for a numerical computation.

(v) What do you mean by modified Trapezoidal rule?

(vi) Give the general expression of modified Euler's method?

(vii) Write down the error term generated in Trapezoidal rule?

(viii) Discuss the types of finite differences by giving at least one example to each method.

(ix) What do you mean by iterative method and discuss the stability criteria of iterative schemes?

(x) What is the difference between Bisection method and the method of false position?

Subjective Part (16*5 = 80 Marks)

Section-I

Q.2. Use Bisection method to find out the root of the function describing the drag coefficient of parachutist given by

$$f(c) = \frac{667.38}{c}\left(1 - \exp(-0.146843c)\right) - 40$$

where $c = 12$ to $c = 16$. Perform at least two iterations.

Q.3. Find out the highest Eigen value and the corresponding Eigen vector for the following matrix

$$A = \begin{bmatrix} 3.556 & -1.778 & 0 \\ -1.778 & 3.556 & -1.778 \\ 0 & -1.778 & 3.556 \end{bmatrix}$$

Use the vector $(1.778,\ 0,\ 1.778)^T$ as an initial guess.

Q.4. Find the solution of the following system of Eqs. by using Gauss-Seidal iteration method for

$$x_1 - \frac{1}{4}x_2 - \frac{1}{4}x_3 = \frac{1}{2}$$

$$-\frac{1}{4}x_1 + x_2 - \frac{1}{4}x_4 = \frac{1}{2}$$

$$-\frac{1}{4}x_1 + x_3 - \frac{1}{4}x_4 = \frac{1}{4}$$

$$-\frac{1}{4}x_2 - \frac{1}{4}x_3 + x_4 = \frac{1}{4}$$

Calculate the first three iterations.

Section-II

Q.5. Prove that $\int_a^b f(x)dx = (b-a)\frac{f(a)+f(b)}{2}$ and use Simpson's 3/8 rule to find the value of the function $y(80)$ for the given data

X	0	10	20	30	40	50	60	70	80
y	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

Q.6. Use Newton's Quadratic interpolation formula to calculate the value of the function at $x = \frac{2\pi}{3}$ when

$f(x) = \frac{x + \cos x}{2}; \quad -\pi \le x \le \pi$ Use step size as $\frac{\pi}{4}$.

Q.7. Using the Taylor's Series method, find the solution of the initial value problem

$$\frac{dy}{dt} = t + y; \qquad y(1) = 0$$

at $t = 1.2$ with $h = 0.1$ and compare the result with closed form solution (exact solution).

Section-III

Q.8. Use the concept of vector form of Euler's method to solve the given differential Eq. by calculating the first two iterations when

$$\frac{d^2y}{dx^2} + 0.5\frac{dy}{dx} + 7y = 0$$

Subjected to $y(0) = 4$ and $y'(0) = 0$, where the value of step size is equal to 0.5.

Q.9. Use the predictor-corrector method to find the approximate value of $y(2)$ if $y(t)$ is the solution of the Eq.

$$\frac{dy}{dt} = \frac{1}{2}(t + y)$$

where $y(0) = 2$, $y(0.5) = 2.636$, $y(1) = 3.595$ and $y(0) = 4.968$.
