

University of Sargodha

M.A/M.Sc Part- II/Composite, 1st-A/2014

Mathematics: XIII Number Theory

Maximum Marks: 100

Time Allowed: 3 Hours

Note: Objective part is compulsory. Attempt any four questions from subjective part.

Objective Part		
Q. 1	Give short answers.	20
(i)	If $(273, 81) = 3$ then find $< 273, 81 >$.	
(ii)	If p is prime and p/ab where a and b are integers then either p/a or p/b .	
(iii)	Define order of an integer modulo m .	
(iv)	Find the primitive root of 3.	
(v)	Evaluate $(\frac{2}{43})$.	
(vi)	Define Mobius function.	
(vii)	State the necessary condition that the class number of a field is 1.	
(viii)	State the property that is equivalent to unique factorization in $R[\theta]$.	
(ix)	Define symmetric polynomial.	
(x)	Define Norm of α where $\alpha \in R(\theta)$.	
(Subjective Part)		
Note: Attempt any four questions.		
Q. 2	(a) Prove that if z is an integer and $y, z > 0$ then	10
	$\left\lfloor \frac{xy}{z} \right\rfloor = \left\lfloor \frac{x}{yz} \right\rfloor$	
	(b) If $F(n) = \sum_{d n} f(d)$ and F is multiplicative then prove that f is multiplicative.	10
Q. 3	(a) if $(b, c) = 1$ then show that $(a, bc) = (a, b)(a, c)$.	10
	(b) Show that if $n > 1$, then the sum of the positive integers less than n and prime to it is $\frac{n\phi(n)}{2}$, where $\phi(n)$ is Eulers function.	10
Q. 4	(a) State and prove Chinese Remainder Theorem.	10
	(b) Given that 2 is a primitive root of 9, construct a table of indices and use to solve the following congruence	10
	$10x \equiv 8 \pmod{18}$.	
Q. 5	(a) Let $R(\theta)$ be an algebraic number field then prove that every $\alpha \in R(\theta)$ is an algebraic number and every field conjugate of α is also a conjugate of α .	10
	(b) If α is a zero of the monic polynomial with coefficients in \mathbb{Z} , then prove that α is an algebraic integer.	10
Q. 6	(a) Prove that $R\sqrt{-5}$ is not a unique factorization domain.	10
	(b) Prove that the product ideal AB does not depend on the representation chosen for ideals A and B .	10
Q. 7	(a) Prove that the discriminant of the cyclotomic field K_p is	10
	$(-1)^{\frac{p-1}{2}} p^{p-2}$	
	(b) An element of $R[\theta]$ is a unit iff its norm is ± 1 .	10