Subject: Math: IV-VI(iv)/IX-XI(iv) M.A/M.Sc: Part- II / Composite, 1st -A/2011

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University of Sargodha

M.A/M.Sc Part-II / Composite, 1st -A/2011

Math: IV-VI(iv)/IX-XI(iv) Rings & Modules

Maximum Marks: 40 **Fictitious #:** ______

Time Allowed: 45 Min. ***Objective Part*** **Signature of CSO:** ______

Note: **Cutting, Erasing, overwriting and use of Lead Pencil are strictly prohibited. Only first attempt will be considered.**

Q.1 (a) Encircle the correct choice in the following MCQ's **(5)**

(i) A ring which is commutative with identity element and having no zero divisor is calleed

(a) Division Ring (b) Integral domain

(c) Prime Ring (d) Primary ring

(ii) If R & R' be arbitrary ring $\phi : R \to R'$ is ring homomorphism such that $\phi(a) = 0 \ \forall \ a \in R$ then $Ker\phi =$ ----------

(a) R' (b) 0

(c) R (d) None of these

(iii) Degree of zero polynomial is

(a) 1 (b) 0 (c) Not defined (d) 2

(iv) Every R- module is isomorphic to a --------------- of a free R-module

(a) Direct summand (b) Quotient module

(c) Projective module (d) Maximal module

(v) A ring with zero characteristic is

(a) Z (b) Z_2

(c) Z_3 (d) $Z_2 \times Z_3$

(b) Fill in the Blanks **(5)**

(i) A field is a commutative ---------------------- ring.

(ii) In monic polynomial the leading coefficient of the polynomial is ---------- of R.

(iii) If U is an ideal of a ring R then homomorphic image of R = ...

(iv) An R-module M is said to be irreducible if its only submodule are ....

(v) In the ring Z of integers the primary ideals are those of the form ....

(c) **State True or False** **(10)**

(i) In the ring of rational numbers the subring of integers is an ideal.

(ii) If R is commutative then all left and right ideal ideals of R are ideals of R.

(iii) All sub-rings of Z_m are ideals.

(iv) The degree of polynomial $4x^3+2x^2+x+4$ is 4

(v) A finite extension E of F is always algebraic.

(vi) A finitely generated module is always a free modules.

(vii) A submodule of a free module over a ring R is always a free module.

(viii) The rings of integers is not a principal ideal ring

(ix) The ring of integers is not a principal ideal ring.

(x) The ring $Z_6=\{0,1,2,3,4,5\}$ w.r.t. + modulo 6 contains no zero divisor.

Q.2 **SHORT QUESTIONS** **(20)**

Define the following

i) Torsion Module **ii)** Principal Module and principal ideal ring

iii) Algebraic element **iv)** Show that the homomorphic image of a commutative ring is commutative.

v) Euclidean domain **vi)** Primary Ideal

vii) If R is a commutative ring and $a\in R$ then show that $aR=\{ar : r\in R\}$ is a two sided ideal.

viii) Canonical Homomorphism of a Module

ix) Quotient Module

x) Is $1+x^2$ irreducible over the field of real numbers? Justify your answer

University of Sargodha

M.A/M.Sc Part- II/Composite, 1st -A/2011

Math: IV-VI(iv)/IX-XI(iv) **Rings & Modules**

Maximum Marks: 60 **Time Allowed: 2:15 Hours**

Subjective Part

Note: **Attempt any four questions, selecting two question from each section. All questions carry equal marks.**

SECTION I

3(a) Let R be commutative ring with identity prove that an ideal M of R is maximal if and only if R/M is a field. (8)

(b) Let R be a ring with $a^2 = a \ \forall \ a \in R$ prove that (7)

(i) $2a = 0 \ \forall \ a \in R$ (ii) $ab = ba \ \forall \ a \in R$

4(a) If R is integral domain then prove that $R[x]$ the polynomial ring over R is integral domain. (8)

(b) Let R be a ring and $f : R \to R$ be a ring homomorphism. Show that the set $S = \{a \in R : f(a) = a\}$ is a subring of R. (7)

5 Let D be unique factorization Domain, then prove that every irreducible element of $D[x]$ is prime. (15)

SECTION II

6(a) Let R be a Euclidean ring then show that any finitely generated R- module M is the direct sum of a finite number of cyclic submodule. (8)

(b) If A and B are submodule of a module C, then prove that A+B is a submodule of C. (7)

7(a) If M is R-modole and if $r \in R$. Prove that the set $rM = \{rm : m \in M\}$ is an R-module. (8)

(b) It T is a homorphism of M on to N with $K(T) = A$, Prove that N is isomorphic to M/A. (7)

8 Show that there exists a free R-module on any set S. (15)