University of Sargodha

M.A/M.Sc Part-1 / Composite, 2nd -A/2010

Math- IV **Mechanics**

Maximum Marks: 40 ***Objective Part*** **Fictitious #:** _____

Time Allowed: 45 Min. **Signature of CSO:** _____

Note: **Cutting, Erasing, overwriting and use of Lead Pencil are strictly prohibited. Only first attempt will be considered.**

1(a)	Tick the correct option in the following (i) If $\vec{V}$ is irrotational vector function, then $\nabla \times \vec{A} =$ (a) 1 (b) 2 (c) 3 (d) 0 (ii) Scale factors for Cartesian coordinate system are (a) 1,1, 1 (b) 1, 2, 1 (c) 2, 1,1 (d) 1,1,2 (iii) The direction cosines of $x_3 - axis$ are (a) (0,1,0) (b) (0,0,1) (c) (1,0,0) (d) (0,0,0) (iv) Kronecker delta symbols is a Cartesian tensor of rank (a) 1 (b) 2 (c) 3 (d) 4 (v) Physically the rate of change of scalar field at a certain point in normal direction is said to be (a) gradient (b) divergence (c) directional derivative (d) none	05
1(b)	Identify the following as true or False (i) Temperature with in a body is a scalar point function. T / F (ii) Angular velocity vector measures the rate of translation of a rigid body. T / F (iii) Angle between the surfaces at a point is the angle between the normals to the surfaces at that point. T / F (iv) A right handed orthogonal transformation is proper transformation. T / F (v) Laplace operator is the square root of the del operator. T / F (vi) A tensor of order zero is called a scalar invariant T / F (vii) Tensors of different ranks can be added. T / F (viii) Subtraction of tensors is associative. T / F (ix) The most general rigid body motion is the linear motion T / F (x) Physically scalar triple product gives the volume of sphere. T / F	10
1(c)	Fill in the blanks (i) ω is independent of the choice of the _____ (ii) The rotation of a rigid body about a point can be described by using _____angular coordinates. (iii) Time derivatives of Euler's angles represents the _____ (iv) _____ region cam be shrunk to a point. (v) _____ is a vector at point P normal to the surface the $u_1 = c_1$	05

Q.2. Solve the followings: **(20)**

i. If $\nabla\varphi = 2r^4\vec{r}$ *find* φ ii. Calculate the area of the region R bounded by a simple closed curve C. iii. Prove that u_1, u_2, u_3, are orthogonal curvilinear coordinates, then $|\nabla uj| = h_j^{-1}$ iv. If φ is a differentiable scalar point function, then prove that $\nabla\times(\nabla\varphi) = 0$ v. State Gauss divergence theorem. vi. Show that the hoop of mass "m" and radius $a/\sqrt{2}$ is equimomental with circular plate of mass "m" and radius "a". vii. Show that if $\varphi(x, y, z)$ is any solution of Laplace's equation, then show that $\nabla\varphi$ is irrotational. viii. Show that rotational K.E of a rigid body is ½ $\underline{\omega}.\vec{L}$ ix. Calculate body cone semi vertical angle if all components of the angular velocity are same. x. Express linear displacement in terms of the angular displacement.

University of Sargodha

M.A/M.Sc Part-1 / Composite, 2nd -A/2010

Math- IV **Mechanics**

Maximum Marks: 60 **Time Allowed: 2:15 Hours**

Subjective Part

Note: **Attempt any three questions. All questions carry equal marks.**

3	(a) Verify the Stokes theorem for the vector function $\vec{A} = z^2 i + x^2 j + y^2 k$ when S is the surface of the parabolied $z = 4 - x^2 - y^2$,lying above xy-plane	10
	(b) Find the transformation matrix for a rotation of angle $\frac{\pi}{4}$ in the negative sense about $x_1 - axis$.	10
4	(a) Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \frac{1}{1+x^2+y^2+z^2} dzdydx$	10
	(b)Show that the finite rotation of rigid body don't commute , but infinitesimal rotation commute. Also show that sum of angular velocities is also an angular velocity.	10
5	(a) State and prove parallel axis theorem.	10
	(b) Find the moment of inertia of a solid homogeneous cube with edge length 2a about the concurrent axes and also their product of inertia.	10
6	(a) Show that any one of the three principal moments of inertia cannot be greater than the sum of other two.	10
	(b) Find the K.E. and angular momentum in terms of Eulerian angles.	10
7	(a) Derive Euler's equations of motion for a rigid body with a fixed point in a body.	10
	(b) A rigid body is rotating about a fixed point with angular velocity $\underline{\omega}$. Assuming The coordinate axes coincide with the principal axes, If T stands for K.E. and $\underline{G}$ for external torque acting on the body. Show that $\frac{dT}{dt} = \vec{G}.\underline{\omega}$	10