

UNIVERSITY OF THE PUNJAB, LAHORE



Mathematics
Part-II A/2007

Roll No. 027975

Examination:- M.A/M.Sc.

Subject:- Numerical Analysis
Paper :- III

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

Attempt FIVE questions selecting at least TWO questions from each Section.

Section I

Q#1 (a) Derive the Regula-Falsi method for the solution of non-linear equations.

(b) Find a root of the equation $X^3 - 4X^2 + X - 10 = 0$ by Regula-Falsi method correct to 3 decimal places taking $a = 4$, $b = 5$. (10,10)

Q#2 (a) Using Crout's method to find the inverse of co-efficient matrix A for the system of equations

$$2X_1 + 3X_2 + X_3 = 9$$

$$X_1 + 2X_2 + 3X_3 = 6$$

$$3X_1 + X_2 + 2X_3 = 8$$

then solve the system of equations using inverse of A^{-1}

(b) Solve the system of equations

$$10X_1 + X_2 + 2X_3 = 44$$

$$X_1 + 2X_2 + 10X_3 = 61$$

$$2X_1 + 10X_2 + X_3 = 51$$

by Gauss-Seidal Iterative method. (10,10)

Q#3 (a) Prove the following

- (i) The sum of the eigenvalues of a matrix A is equal to the sum of its diagonal elements.
- (ii) The product of the eigenvalues of a matrix A is equal to $\det A$.

(b) Define Dominant Eigenvalue. Find the dominant eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

(5+5,10)

Section II

Q#4 (a) (i) Evaluate $(\Delta^2/E) X^3$

(ii) Show that $\delta^n y_x = \nabla^n E y_x$.

(b) Using Atiken's method find the polynomial passing through (1,4), (3,7), (4,8), (6,11) and hence find value at $X=5$.
(5+5,10)

Q#5 (a) Derive the Stirling formula for central difference interpolation.

(b) Using Striling formula to find $y(28)$ given that $y(20) = 49225$, $y(25) = 48316$, $y(30) = 47236$, $y(35) = 45926$, $y(40) = 44306$.
(10,10)

✓ Q#6 (a) Establish the Gaussian Quadrature for two points.

✓ (b) Evaluate $\int_0^1 (\sin(x)/x) dx$ by Gauss Quadrature.
(10,10)

Section III

Q#7 (a) Form the difference equation given that

$$y_n = A.3^n + B.5^n \text{ where } A, B \text{ are arbitrary constants.}$$

✓ (b) Solve the difference equation $Y_{k+2} - 3 Y_{k+1} + 2Y_k = k^2$.
(10,10)

✓ Q#8 (a) Derive the Modified Euler's method for solution of ordinary differential equations.

✓ (b) Solve $dy/dx = x^2 - y$, $y(0) = 1$ at $x = 0.1, 0.2, 0.3$ and 0.4 by Taylor Series method compare vaues with exact.
(10,10)

✓ Q#9 (a) Given that $dy/dx = (y^2 - x^2) / (y^2 + x^2)$ with $x_0 = 0$, $y_0 = 1$ $h = 0.2$ find y_1 and y_2 using Runge- Kutta method of order four.

(b) The differential eq. $y' + y^2 - x^2 = 0$ with boundary conditions $y = 0$ when $x = 0$ is satisfied by the of x and y in the following table:

X	-0.4	-0.2	0.2	0.4
Y	0.02131	0.00267	-0.00267	-0.02131

Use Adam-Bashforth formula to obtain to 4 dp the value of y when $x = 0.6$.
(10,10)